

Chapter 10 — Squares, Square Roots, and Estimation

Teacher Chapter-at-a-Glance

Materials

A perfect-squares reference chart and number line for estimation are helpful.

Talk Prompt

Display $\sqrt{50}$. Ask what question the symbol asks. Accept only explanations involving a number multiplied by itself, not dividing by 2.

If Ready-to-Learn Reveals a Gap

Rebuild multiplication facts and the link between n^2 and $n \times n$ before moving to roots if Ready-to-Learn reveals a gap.

Mastery Decision Guide

What a missed Tripwire item actually means, and what to do about it before moving on.

Missed Tripwire #1 → Treating square root as halving (Mistake 1). *A student divides the radicand by 2 signals this error.* Review before the next chapter.

Missed Tripwire #2 → Choosing wrong benchmarks (Mistake 4). *An interval wider than one whole number signals this error.* Review before the next chapter.

Standards Alignment

This book is designed as a Number Sense foundation and bridge, deliberately spanning the grades where students most often arrive at grades 7-9 without having mastered the material. Each chapter below is tagged with the grade level where its content is typically taught, so any teacher can place it against whatever curriculum they use, without needing to recognize a specific country's standard codes.

1. Place Value, Number Forms, and Comparing

G4–G5 — Place value, decimal comparison, rounding

2. Factors, Multiples, Primes, and Divisibility

G6 — GCF, LCM, prime factorization

3. Fraction Foundations

G4–G5 — Equivalence, comparison, fraction forms

4. Decimal Sense and Decimal Operations

G6 — Add, subtract, multiply, divide decimals

5. Integer Sense

G6 — Integers, number line, absolute value

6. Integer Operations

G7 — Add, subtract, multiply, divide integers

7. Rational Numbers

G7 — Rational number system, mixed-form comparison

8. Order of Operations

G6 — PEMDAS, left-to-right priority, brackets

9. Powers and Exponents

G6–G8 — Exponent meaning, same-base rules

10. Squares, Square Roots, Estimation

G8 — Perfect squares, root estimation

Ready-to-Learn Check

Quick check before starting. A wrong answer below points to a specific gap worth closing first, not just a miss to shrug off.

1. Calculate: 8×8

Correct answer: 64

Likely wrong answer: 16

Misconception: Needs reinforcement with multiplication facts and repeated factors.

Next move: Write 8×8 as eight groups of 8 or use a known fact.

2. Evaluate: 6^2

Correct answer: 36

Likely wrong answer: 12

Misconception: Treats an exponent as ordinary multiplication.

Next move: Expand 6^2 as 6×6 .

3. Which whole number is between 6 and 8?

Correct answer: 7

Likely wrong answer: 14

Misconception: Needs reinforcement with number-line order.

Next move: Mark 6, 7, and 8 on a number line.

4. Which value is closer to 100: 91 or 109?

Correct answer: They are equally close.

Likely wrong answer: 91

Misconception: Compares size rather than distance from the benchmark.

Next move: Find each distance from 100: both are 9 away.

Common Mistakes

Students often run into the same kinds of trouble with this topic. Recognizing these patterns helps avoid them.

Comparing 0.86 and 0.809 by Place Value

	Ones		Tenths	Hundredths	Thousandths
0.86	0	.	8	6	0
0.809	0	.	8	0	9

Ones tie. Tenths tie. Hundredths decide it: $6 > 0$, so $0.86 > 0.809$.

$\sqrt{64} = 8$ gives one answer; $x^2 = 64$ allows two.

Mistake 1: Treating square root as halving

What students do: Evaluating $\sqrt{50}$ as 25 because $50 \div 2 = 25$.

Why this happens: Students assume the inverse of squaring is dividing by 2.

What to do instead: Ask for equal factors: what number, multiplied by itself, gives this value?

Diagnostic trigger: A student divides the radicand by 2 signals this error.

Mistake 2: Using any factor pair

What students do: Stating $\sqrt{24} = 6$ because $4 \times 6 = 24$.

Why this happens: Students remember multiplication but forget the factors must be equal.

What to do instead: A square root needs a number multiplied by itself, not any two factors.

Diagnostic trigger: A student uses unequal factors signals this error.

Mistake 3: Thinking every root is whole

What students do: Stating $\sqrt{90}$ has no answer because 90 is not a perfect square.

Why this happens: Early examples often use perfect squares, making non-perfect roots feel unsolvable.

What to do instead: Estimate non-perfect roots between the two nearest perfect squares.

Diagnostic trigger: A student says a non-perfect root is impossible signals this error.

Mistake 4: Choosing wrong benchmarks

What students do: Estimating $\sqrt{55}$ using 25 and 100 instead of the closest perfect squares.

Why this happens: The student does not locate the perfect squares closest to the number.

What to do instead: Find the nearest perfect square below and the nearest perfect square above.

Diagnostic trigger: An interval wider than one whole number signals this error.

Mistake 5: Giving two answers for a radical

What students do: Writing $\sqrt{64} = \pm 8$.

Why this happens: Students confuse a radical expression with solving $x^2 = 64$.

What to do instead: A radical gives the principal positive root only, never a $\pm$ pair.

Diagnostic trigger: A student includes $\pm$ after a radical signals this error.

Practice

Teacher solutions follow this practice set.

1. Evaluate: $\sqrt{25}$
2. Evaluate: $\sqrt{121}$
3. Evaluate: $\sqrt{169}$
4. Is 98 a perfect square? Explain.
5. Is 225 a perfect square? Explain.
6. Estimate: $\sqrt{30}$
7. Estimate: $\sqrt{55}$
8. Estimate: $\sqrt{95}$
9. Is $\sqrt{180}$ closer to 13 or 14?
10. Is $\sqrt{240}$ closer to 15 or 16?
11. A square has an area of 324 cm^2 . Find the side length.
12. Explain why $\sqrt{72}$ cannot equal 36.

Practice Solutions

Use these concise solutions to check the method as well as the final answer.

1. Evaluate: $\sqrt{25}$

Solution: $\sqrt{25} = 5$ because $5^2 = 25$.

2. Evaluate: $\sqrt{121}$

Solution: $\sqrt{121} = 11$ because $11^2 = 121$.

3. Evaluate: $\sqrt{169}$

Solution: $\sqrt{169} = 13$ because $13^2 = 169$.

4. Is 98 a perfect square? Explain.

Solution: No. $9^2 = 81$ and $10^2 = 100$, so 98 lies between consecutive perfect squares.

5. Is 225 a perfect square? Explain.

Solution: Yes. $15^2 = 225$.

6. Estimate: $\sqrt{30}$

Solution: $25 < 30 < 36$, so $5 < \sqrt{30} < 6$. To the nearest tenth, $\sqrt{30} \approx 5.5$.

7. Estimate: $\sqrt{55}$

Solution: $49 < 55 < 64$, so $7 < \sqrt{55} < 8$. To the nearest tenth, $\sqrt{55} \approx 7.4$.

8. Estimate: $\sqrt{95}$

Solution: $81 < 95 < 100$, so $9 < \sqrt{95} < 10$. To the nearest tenth, $\sqrt{95} \approx 9.7$.

9. Is $\sqrt{180}$ closer to 13 or 14?

Solution: $13^2 = 169$ and $14^2 = 196$. Since $\sqrt{180} \approx 13.4$, it is closer to 13.

10. Is $\sqrt{240}$ closer to 15 or 16?

Solution: $15^2 = 225$ and $16^2 = 256$. Since $\sqrt{240} \approx 15.5$ but is slightly below 15.5, it is closer to 15.

11. A square has an area of 324 cm^2 . Find the side length.

Solution: The side length is $\sqrt{324} = 18 \text{ cm}$.

12. Explain why $\sqrt{72}$ cannot equal 36.

Solution: A square root is a number that squares to the radicand. Since $36^2 = 1,296$, $\sqrt{72}$ cannot equal 36; in fact, $8 < \sqrt{72} < 9$.

Tripwire Check

These questions test whether the thinking is solid — not just whether the procedure was memorized. Each wrong answer below points to a specific misconception, not just "try again."

1. A student says $\sqrt{98} = 49$ because $98 \div 2 = 49$. Explain the error. Then estimate $\sqrt{98}$.

Correct answer: A square root does not mean divide by 2. $9^2 < 98 < 10^2$, so $9 < \sqrt{98} < 10$, approximately 9.9.

Likely wrong answer: 49

Misconception: Treats square root as halving.

Next move: Return to Worked Example 3 and ask what equal factors would produce the radicand.

2. A student estimates $\sqrt{155}$ by writing $10 < \sqrt{155} < 14$. Explain why this interval is too wide. Then give the correct interval and a reasonable estimate.

Correct answer: $12^2 = 144$ and $13^2 = 169$, so $12 < \sqrt{155} < 13$, approximately 12.4.

Likely wrong answer: $10 < \sqrt{155} < 14$

Misconception: Does not locate consecutive perfect-square benchmarks.

Next move: Find the closest perfect square below and above the radicand.

Answer Key — Worksheets (Full Diagnostic)

Worksheet A: Guided Practice

1. **Evaluate:** $\sqrt{4}$

1. 2. Likely wrong answer: $2 = 4 \div 2$ (halves the radicand instead of finding equal factors). Misconception: A square root is treated as dividing the radicand by 2 instead of finding a number that multiplies by itself to give it.

2. **Evaluate:** $\sqrt{9}$

2. 3. Likely wrong answer: $4.5 = 9 \div 2$ (halves the radicand instead of finding equal factors). Misconception: A square root is treated as dividing the radicand by 2 instead of finding a number that multiplies by itself to give it.

3. **Is 361 a perfect square? Explain.**

3. Yes. $19^2 = 361$. Likely wrong answer: No, without actually testing a nearby whole number. Misconception: A perfect-square claim is judged without checking whether any whole number actually squares to the target.

4. **Evaluate:** $\sqrt{16}$

4. 4. Likely wrong answer: $8 = 16 \div 2$ (halves the radicand instead of finding equal factors). Misconception: A square root is treated as dividing the radicand by 2 instead of finding a number that multiplies by itself to give it.

5. Evaluate: $\sqrt{289}$

5. 17. Likely wrong answer: uses an unequal factor pair that happens to multiply to 289. Misconception: A factor pair is accepted without checking that both factors are actually equal to each other.

6. Is 210 a perfect square? Explain.

6. No. It lies between 196 and 225. Likely wrong answer: Yes, without checking against actual nearby perfect squares. Misconception: A perfect-square claim is judged without checking whether any whole number actually squares to the target.

7. Estimate: $\sqrt{74}$

7. 8.6. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 74. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

8. Estimate: $\sqrt{115}$

8. 10.7. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 115. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

9. Estimate: $\sqrt{137}$

9. 11.7. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 137. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

10. Between which two whole numbers is $\sqrt{175}$?

10. Between 13 and 14. Likely wrong answer: names a non-consecutive pair of whole numbers. Misconception: The interval given does not consist of two consecutive whole numbers.

11. Is $\sqrt{215}$ closer to 14 or 15?

11. Closer to 15. Likely wrong answer: 14 (compares distances to the benchmarks incorrectly). Misconception: The distance from the radicand to each nearby perfect square is compared incorrectly.

12. Is $\sqrt{275}$ closer to 16 or 17?

12. Closer to 17. Likely wrong answer: 16 (compares distances to the benchmarks incorrectly). Misconception: The distance from the radicand to each nearby perfect square is compared incorrectly.

13. A square has area 529 cm^2 . Find the side length.

13. 23 cm. Likely wrong answer: 264.5 cm (halves the area instead of finding its square root). Misconception: The area is divided by 2 instead of having its square root taken to find the side length.

14. A square garden has area 576 m^2 . Find the side length.

14. 24 m. Likely wrong answer: 288 m (halves the area instead of finding its square root). Misconception: The area is divided by 2 instead of having its square root taken to find the side length.

15. Explain why $\sqrt{400}$ is 20, not -20.

15. The radical symbol gives the principal (positive) square root: $\sqrt{400} = 20$. Likely wrong answer: says both 20 and -20 are correct answers to the radical expression.

Misconception: The radical symbol is not recognized as specifying the positive root only, unlike solving an equation such as $x^2 = 400$.

Worksheet B: Independent Practice

1. Evaluate: $\sqrt{1}$

1. 1. Likely wrong answer: $0.5 = 1 \div 2$ (halves the radicand instead of finding equal factors). Misconception: A square root is treated as dividing the radicand by 2 instead of finding a number that multiplies by itself to give it.

2. Evaluate: $\sqrt{484}$

2. 22. Likely wrong answer: $242 = 484 \div 2$ (halves the radicand instead of finding equal factors). Misconception: A square root is treated as dividing the radicand by 2 instead of finding a number that multiplies by itself to give it.

3. Is 961 a perfect square? Explain.

3. Yes. $31^2 = 961$. Likely wrong answer: No, without actually testing a nearby whole number. Misconception: A perfect-square claim is judged without checking whether any whole number actually squares to the target.

4. Evaluate: $\sqrt{625}$

4. 25. Likely wrong answer: $312.5 = 625 \div 2$ (halves the radicand instead of finding equal factors). Misconception: A square root is treated as dividing the radicand by 2 instead of finding a number that multiplies by itself to give it.

5. Evaluate: $\sqrt{729}$

5. 27. Likely wrong answer: uses an unequal factor pair that happens to multiply to 729. Misconception: A factor pair is accepted without checking that both factors are actually equal to each other.

6. Evaluate: $\sqrt{841}$

6. 29. Likely wrong answer: uses an unequal factor pair that happens to multiply to 841. Misconception: A factor pair is accepted without checking that both factors are actually equal to each other.

7. Is 510 a perfect square? Explain.

7. No. It lies between 484 and 529. Likely wrong answer: Yes, without checking against actual nearby perfect squares. Misconception: A perfect-square claim is judged without checking whether any whole number actually squares to the target.

8. Estimate: $\sqrt{15}$

8. 3.9. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 15. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

9. Estimate: $\sqrt{42}$

9. 6.5. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 42. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

10. Between which two whole numbers is $\sqrt{315}$?

10. Between 17 and 18. Likely wrong answer: names a non-consecutive pair of whole numbers. Misconception: The interval given does not consist of two consecutive whole numbers.

11. Estimate: $\sqrt{88}$

11. 9.4. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 88. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

12. Estimate: $\sqrt{185}$

12. 13.6. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 185. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

13. Estimate: $\sqrt{235}$

13. 15.3. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 235. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

14. Between which two whole numbers is $\sqrt{470}$?

14. Between 21 and 22. Likely wrong answer: names a non-consecutive pair of whole numbers. Misconception: The interval given does not consist of two consecutive whole numbers.

15. Is $\sqrt{360}$ closer to 18 or 19?

15. Closer to 19. Likely wrong answer: 18 (compares distances to the benchmarks incorrectly). Misconception: The distance from the radicand to each nearby perfect square is compared incorrectly.

16. Is $\sqrt{455}$ closer to 21 or 22?

16. Closer to 21. Likely wrong answer: 22 (compares distances to the benchmarks incorrectly). Misconception: The distance from the radicand to each nearby perfect square is compared incorrectly.

17. A square has area 676 m^2 . Find the side length.

17. 26 m. Likely wrong answer: 338 m (halves the area instead of finding its square root). Misconception: The area is divided by 2 instead of having its square root taken to find the side length.

18. A square tile has area 784 cm^2 . Find the side length.

18. 28 cm. Likely wrong answer: 392 cm (halves the area instead of finding its square root). Misconception: The area is divided by 2 instead of having its square root taken to find the side length.

19. Explain why $\sqrt{135}$ is not 67.5.

19. A square root seeks equal factors, not half of the radicand. Likely wrong answer: says it's close enough since 67.5 seems reasonable. Misconception: The explanation doesn't identify the actual error (halving instead of finding equal factors).

20. Explain why $\sqrt{900}$ is not ± 30 .

20. A radical gives the positive principal root only: $\sqrt{900} = 30$. Likely wrong answer: says ± 30 is the correct radical answer. Misconception: The radical symbol is not recognized as specifying the positive root only.

Worksheet C: Mixed Application

1. A student says $\sqrt{36} = 18$ because $36 \div 2 = 18$. Explain the mistake and find the correct value.

1. 6. A square root asks for equal factors, not half.

2. A student says $\sqrt{270} = 15$ because $15 \times 18 = 270$. Explain the mistake and estimate the root correctly.

2. 15 and 18 are not equal factors. $16^2 = 256$ and $17^2 = 289$, so $16 < \sqrt{270} < 17$, approximately 16.4.

3. Explain why $\sqrt{1024} = 32$, but $x^2 = 1024$ has two solutions.

3. $\sqrt{1024} = 32$ gives the principal positive root only. $x^2 = 1024$ allows $x = 32$ or $x = -32$, since both square to 1024. Likely wrong answer: says both should have one answer, or both should have two. Misconception: The distinction between a radical expression's single principal root and an equation's two solutions is not recognized.

4. Estimate: $\sqrt{112}$

4. 10.6. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 112. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

5. Estimate: $\sqrt{148}$

5. 12.2. Likely wrong answer: uses benchmarks that are not the nearest perfect squares to 148. Misconception: The estimate is built from perfect squares that are not the closest ones to the target number.

6. Which is larger: $\sqrt{220}$ or 15?

6. 15 is larger ($\sqrt{220} \approx 14.8$). Likely wrong answer: says $\sqrt{220}$ is larger without estimating it first. Misconception: A comparison between a radical and a whole number is made without actually estimating the radical's value.

7. Which is larger: 18 or $\sqrt{340}$?

7. $\sqrt{340}$ is larger (≈ 18.4). Likely wrong answer: says 18 is larger without estimating the radical first. Misconception: A comparison between a whole number and a radical is made without actually estimating the radical's value.

8. A square garden has area 1089 m^2 . Find the side length.

8. 33 m. Likely wrong answer: 544.5 m (halves the area instead of finding its square root). Misconception: The area is divided by 2 instead of having its square root taken to find the side length.

9. A square tile has area 1156 cm^2 . Find the side length.

9. 34 cm. Likely wrong answer: 578 cm (halves the area instead of finding its square root). Misconception: The area is divided by 2 instead of having its square root taken to find the side length.

10. Explain why $\sqrt{150}$ cannot be 75.

10. $75 \times 75 = 5625$, not 150. A square root needs equal factors of 150 itself, not half of it. Likely wrong answer: says 75 is close enough to be acceptable. Misconception: The explanation doesn't verify the claim by checking what 75 actually squares to.

11. Order from least to greatest: $\sqrt{48}$, 8, $\sqrt{12}$, $\sqrt{28}$

11. $\sqrt{12}$, $\sqrt{28}$, $\sqrt{48}$, 8. Likely wrong answer: orders using the numbers under the radicals directly, ignoring that 8 is not under a radical. Misconception: A whole number and several radicals are compared without converting the whole number to an equivalent radical or estimating each radical first.

12. Order from least to greatest: $\sqrt{375}$, $\sqrt{265}$, $\sqrt{335}$, $\sqrt{300}$

12. $\sqrt{265}$, $\sqrt{300}$, $\sqrt{335}$, $\sqrt{375}$. Likely wrong answer: orders using the radicands as if they scale directly with the roots' spacing. Misconception: Radicals are ordered by radicand size assumptions rather than by actually estimating each root.

13. A square has perimeter 116 cm. Find its area.

13. Side = 29 cm; area = 841 cm^2 . Likely wrong answer: 841 cm^2 directly from the perimeter, skipping the step of finding the side length first. Misconception: The area is calculated without first finding the side length from the perimeter.

14. Non-Routine: Choose a number between 225 and 256 that has not appeared earlier. Estimate its square root and explain the interval.

14. Answers vary. Example: $225 < 240 < 256$, so $15 < \sqrt{240} < 16$; $\sqrt{240}$ is about 15.5. Likely wrong answer: gives a value outside 15 to 16. Misconception: Consecutive perfect-square bounds are not used. Next move: Write 15^2 and 16^2 before estimating.

15. Non-Routine: Create a square whose area is a perfect square between 900 and 1156 that has not appeared earlier. State the area and find the side length.

15. Answers vary. Example: area $1,024 \text{ cm}^2$ gives side length $\sqrt{1,024} = 32 \text{ cm}$. Likely wrong answer: chooses a non-perfect-square area. Misconception: The area is not checked as the square of a whole-number side. Next move: List 31^2 , 32^2 , and 33^2 and select a value inside the interval.

Answer Key — Exit Ticket

1. 16.
2. 11.4.
3. 21 m.