

Chapter 7 — Rational Numbers

Teacher Chapter-at-a-Glance

Materials

A number line, printed or drawn on the board. No other materials required.

Talk Prompt

Ask the class which is greater, $-\frac{3}{4}$ or -0.7 , before teaching the method. Let the room split before converting both to decimals.

If Ready-to-Learn Reveals a Gap

If two or more Ready-to-Learn items reveal a gap, stop and rebuild fraction simplification and decimal comparison before moving into mixed-form comparisons. Both are used constantly in this chapter.

Mastery Decision Guide

What a missed Tripwire item actually means, and what to do about it before moving on.

Missed Tripwire #1 → Comparing negative rationals like positive numbers (Mistake 4). *A comparison between a fraction and a decimal with no conversion step shown signals this error.*
Review before the next chapter.

Missed Tripwire #2 → Thinking every decimal is rational (Mistake 3). *A non-repeating, non-terminating decimal classified as rational signals this error.* Review before the next chapter.

Ready-to-Learn Check

Quick check before starting. A wrong answer below points to a specific gap worth closing first, not just a miss to shrug off.

1. Simplify: $\frac{9}{12}$

Correct answer: $\frac{3}{4}$

Likely wrong answer: $\frac{9}{12}$

Misconception: Divides by a common factor that isn't the GCF, or doesn't simplify at all.

Next move: Have the student find the GCF of 9 and 12 before dividing.

2. Compare: $0.6 \underline{\hspace{1cm}} 0.58$

Correct answer: $0.6 > 0.58$

Likely wrong answer: $0.6 < 0.58$

Misconception: Assumes more digits after the decimal point means a bigger number.

Next move: Have the student line up both numbers at the decimal point before comparing.

3. Compare: $-8 \underline{\hspace{1cm}} -3$

Correct answer: $-8 < -3$

Likely wrong answer: $-8 > -3$

Misconception: Assumes a bigger digit makes a bigger negative number.

Next move: Have the student picture both on a number line before answering.

Common Mistakes

Students often run into the same kinds of trouble with this topic. Recognizing these patterns helps avoid them.

Comparing 0.86 and 0.809 by Place Value

	Ones		Tenths	Hundredths	Thousandths
0.86	0	.	8	6	0
0.809	0	.	8	0	9

Ones tie. Tenths tie. Hundredths decide it: $6 > 0$, so $0.86 > 0.809$.

Different forms name the same points on the number line.

Mistake 1: Thinking rational means "reasonable"

What students do: Assuming the math word "rational" relates to the everyday word "reasonable."

Why this happens: The two words look and sound alike, and the everyday meaning is far more familiar.

What to do instead: In math, rational specifically means "can be written as one integer over another." It has nothing to do with whether a number seems reasonable.

Diagnostic trigger: An explanation of "rational" that references common sense rather than the integer-over-integer definition signals this error.

Mistake 2: Forgetting integers are rational

What students do: Saying a whole number like 7 is not rational because it "isn't a fraction."

Why this happens: Rational numbers are so strongly associated with fractions that whole numbers can feel like they belong to a separate category.

What to do instead: Every integer can be written over 1. That alone makes it rational.

Diagnostic trigger: An integer marked "not rational" signals this error immediately.

Mistake 3: Thinking every decimal is rational

What students do: Assuming any decimal, including a non-repeating, non-terminating one, must be rational.

Why this happens: Every decimal LOOKS the same on the page, so it's easy to assume they all behave the same way underneath.

What to do instead: Terminating and repeating decimals are rational. A decimal that goes on forever with no repeating pattern is irrational, and cannot be written as a fraction.

Diagnostic trigger: A non-repeating, non-terminating decimal classified as rational signals this error.

Mistake 4: Comparing negative rationals like positive numbers

What students do: Comparing $-\frac{3}{5}$ and -0.55 by digit size instead of converting to the same form.

Why this happens: The forms look different enough that a same-form comparison feels unnecessary, especially under time pressure.

What to do instead: Convert both values to the same form first. Always. Digit size across two different forms means nothing on its own.

Diagnostic trigger: A comparison between a fraction and a decimal with no conversion step shown signals this error.

Mistake 5: Changing form but changing value by accident

What students do: Converting a fraction to a decimal (or back) and landing on a different value than the original.

Why this happens: A rushed division or an incomplete place-value read can silently shift the value during the rewrite.

What to do instead: Check the conversion by converting back. If the round trip doesn't return the original value, the conversion has an error somewhere.

***Diagnostic trigger:** A converted value that doesn't match the original when converted back signals this error.*

Practice

Teacher solutions follow this practice set.

1. Is 9 rational?
2. Is $-\frac{4}{7}$ rational?
3. Is 0.125 rational?
4. Write 0.8 as a simplified fraction.
5. Write -0.35 as a simplified fraction.
6. Write $\frac{7}{8}$ as a decimal.
7. Compare: $\frac{3}{4}$ ____ 0.7
8. Compare: -0.45 ____ $-\frac{4}{9}$
9. Order from least to greatest: $-\frac{1}{4}$, -0.4, 0, $\frac{3}{10}$
10. Calculate: $\frac{1}{4} + 0.5$

Practice Solutions

Use these concise solutions to check the method as well as the final answer.

1. Is 9 rational?

Solution: Yes. $9 = 9/1$, so it is a ratio of integers.

2. Is $-\frac{4}{7}$ rational?

Solution: Yes. $-4/7$ is already a ratio of integers with a nonzero denominator.

3. Is 0.125 rational?

Solution: Yes. 0.125 terminates and equals $125/1000 = 1/8$.

4. Write 0.8 as a simplified fraction.

Solution: $0.8 = 8/10 = 4/5$.

5. Write -0.35 as a simplified fraction.

Solution: $-0.35 = -35/100 = -7/20$.

6. Write $\frac{7}{8}$ as a decimal.

Solution: $7/8 = 7 \div 8 = 0.875$.

7. Compare: $\frac{3}{4}$ ____ 0.7

Solution: $3/4 = 0.75$, and $0.75 > 0.7$. Therefore $3/4 > 0.7$.

8. Compare: -0.45 ____ $-\frac{4}{9}$

Solution: $-4/9 \approx -0.444...$, which is closer to zero than -0.45 . Therefore $-0.45 < -4/9$.

9. Order from least to greatest: $-\frac{1}{4}$, -0.4 , 0 , $\frac{3}{10}$

Solution: Convert to decimals: $-1/4 = -0.25$ and $3/10 = 0.3$. Least to greatest: -0.4 , $-1/4$, 0 , $3/10$.

10. Calculate: $\frac{1}{4} + 0.5$

Solution: $1/4 + 0.5 = 0.25 + 0.50 = 0.75 = 3/4$.

Tripwire Check

These questions test whether the thinking is solid — not just whether the procedure was memorized. Each wrong answer below points to a specific misconception, not just "try again."

1. Compare: $-\frac{2}{5}$ ____ -0.35

Correct answer: -0.35 . $-\frac{2}{5} = -0.4$, and -0.35 is closer to zero.

Likely wrong answer: $-\frac{2}{5}$, from comparing the digits $2/5$ and 0.35 directly without converting.

Misconception: Compares a fraction and a decimal without converting to the same form first.

Next move: Have the student convert the fraction to a decimal before comparing anything.

2. Is $0.333\ldots$ (repeating) rational?

Correct answer: Yes. It equals $\frac{1}{3}$, a fraction of two integers.

Likely wrong answer: No, since it never terminates.

Misconception: Assumes any decimal that goes on forever must be irrational, without checking whether it repeats.

Next move: Have the student distinguish repeating (rational) from non-repeating, non-terminating (irrational) decimals explicitly.

Answer Key — Worksheets (Full Diagnostic)

Worksheet A: Guided Practice

1. Is 4 rational?

1. Yes ($4/1$). Likely wrong answer: No (assumes only fractions or decimals can be rational). Misconception: A whole number is not recognized as rational because it isn't already written as a fraction.

2. Is -7 rational?

2. Yes ($-7/1$). Likely wrong answer: No (assumes negative whole numbers can't be rational). Misconception: A negative whole number is not recognized as rational because it isn't already written as a fraction.

3. Is 0.25 rational?

3. Yes (it terminates). Likely wrong answer: No (assumes any decimal that isn't a whole number can't be rational). Misconception: A terminating decimal is not recognized as expressible as a fraction of two integers.

4. Write 0.5 as a fraction.

4. $\frac{1}{2}$. Likely wrong answer: $\frac{5}{10}$ left unsimplified as the final answer, or $\frac{5}{1}$.
Misconception: Place value is misread, or the fraction is left in an unsimplified form when a simplified one was intended.

5. Is $\frac{1}{0}$ rational?

5. No (division by zero is undefined). Likely wrong answer: Yes (assumes any fraction-looking expression is automatically rational). Misconception: Division by zero is not recognized as making an expression undefined rather than rational.

6. Write 0.25 as a fraction.

6. $\frac{1}{4}$. Likely wrong answer: $\frac{25}{100}$ left unsimplified as the final answer. Misconception: The fraction is correctly built from place value but not reduced to lowest terms.

7. Write -0.6 as a fraction.

7. $-\frac{3}{5}$. Likely wrong answer: $\frac{6}{10}$ or $\frac{3}{5}$ without the negative sign carried through. Misconception: The negative sign from the original decimal is dropped when converting to a fraction.

8. Write $\frac{1}{2}$ as a decimal.

8. 0.5. Likely wrong answer: 0.2 or 0.12 (misplaces the decimal point during division). Misconception: The division of numerator by denominator is set up with the decimal point in the wrong position.

9. Write $\frac{3}{4}$ as a decimal.

9. 0.75. Likely wrong answer: 0.34 (divides in the wrong direction or misplaces the decimal). Misconception: The numerator and denominator are divided in the wrong order, or the decimal point is misplaced.

10. Compare 0.5 and $\frac{1}{2}$.

10. Equal: $0.5 = \frac{1}{2}$. Likely wrong answer: says they are different because one looks like a fraction and one doesn't. Misconception: Two forms of the same value are judged unequal based on appearance instead of actual value.

11. Compare -0.4 and $-\frac{2}{5}$.

11. Equal: $-0.4 = -2/5$. Likely wrong answer: says they are different because one looks like a fraction and one doesn't. Misconception: Two forms of the same value are judged unequal based on appearance instead of actual value.

12. Order from least to greatest: -0.5 , 0 , $1/4$.

12. -0.5 , 0 , $1/4$. Likely wrong answer: orders by treating the fraction as automatically larger since it 'looks positive'. Misconception: A fraction and decimals are compared without converting to the same form first.

13. Calculate: $0.25 + 0.5$

13. $3/4 = 0.75$. Likely wrong answer: 0.75 left as a decimal, or an incorrect fraction is given as the equivalent. Misconception: A decimal sum is not connected back to fraction form when a fraction was requested.

14. Calculate: -0.5×4

14. $-0.5 \times 4 = -2$. Likely wrong answer: 2 (drops the negative sign in the product). Misconception: A negative number multiplied by a positive is not carried through with the correct sign.

15. Explain why $0.75 = 3/4$.

15. $0.75 = 75/100 = 3/4$ after simplifying by GCF 25 . Likely wrong answer: simplifies $75/100$ incorrectly, or doesn't find the GCF needed. Misconception: The fraction is reduced using a factor that isn't the greatest common factor, or the reduction is skipped.

Worksheet B: Independent Practice

1. Is -12 rational?

1. Yes. Likely wrong answer: No (assumes negative whole numbers can't be rational). Misconception: A negative whole number is not recognized as rational because it isn't already written as a fraction.

2. Is 0.9 rational?

2. Yes. Likely wrong answer: No (assumes only whole numbers or fractions can be rational). Misconception: A terminating decimal is not recognized as expressible as a fraction of two integers.

3. Is $5/0$ rational?

3. No (division by zero is undefined). Likely wrong answer: Yes (assumes any fraction-looking expression is automatically rational). Misconception: Division by zero is not recognized as making an expression undefined rather than rational.

4. Write 0.15 as a simplified fraction.

4. $\frac{3}{20}$. Likely wrong answer: $\frac{15}{100}$ left unsimplified as the final answer. Misconception: The fraction is correctly built from place value but not reduced to lowest terms.

5. Is $-\frac{6}{11}$ rational?

5. Yes. Likely wrong answer: No (assumes a fraction with a negative sign can't be rational). Misconception: The presence of a negative sign is mistaken for disqualifying a fraction from being rational.

6. Write 0.72 as a simplified fraction.

6. $\frac{18}{25}$. Likely wrong answer: $\frac{72}{100}$ left unsimplified as the final answer. Misconception: The fraction is correctly built from place value but not reduced to lowest terms.

7. Write -0.05 as a simplified fraction.

7. $-\frac{1}{20}$. Likely wrong answer: $\frac{5}{100}$ or $\frac{1}{20}$ without the negative sign carried through. Misconception: The negative sign from the original decimal is dropped when converting to a fraction.

8. Write $\frac{3}{5}$ as a decimal.

8. 0.6. Likely wrong answer: 0.35 or 0.53 (misplaces the decimal point or divides in the wrong direction). Misconception: The division of numerator by denominator is set up incorrectly.

9. Write $-\frac{7}{8}$ as a decimal.

9. -0.875 . Likely wrong answer: 0.875 without the negative sign carried through. Misconception: The negative sign from the original fraction is dropped when converting to a decimal.

10. Write $\frac{9}{20}$ as a decimal.

10. 0.45. Likely wrong answer: 0.9/20 or 4.5 (misplaces the decimal point during division). Misconception: The division of numerator by denominator is set up with the decimal point in the wrong position.

11. Compare: 0.3 ____ $\frac{1}{3}$

11. $\frac{1}{3}$ ($0.3333 > 0.3$). Likely wrong answer: 0.3 (compares without converting the fraction to a matching decimal form first). Misconception: A fraction and a decimal are compared by appearance instead of converting to the same form.

12. Compare: -0.7 ____ $-\frac{5}{8}$

12. $-\frac{5}{8}$ ($-0.625 > -0.7$). Likely wrong answer: -0.7 (compares negative values as if the larger digit means the greater value). Misconception: A bigger digit on a negative decimal or fraction is mistaken for a bigger value.

13. Compare: $\frac{7}{10}$ ____ 0.71

13. 0.71 ($0.71 > 0.7$). Likely wrong answer: $\frac{7}{10}$ (compares without converting the fraction to a matching decimal form first). Misconception: A fraction and a decimal are compared by appearance instead of converting to the same form.

14. Order from least to greatest: -0.3 , $\frac{1}{5}$, 0, 0.4.

14. -0.3 , 0, $\frac{1}{5}$, 0.4. Likely wrong answer: orders by treating the fraction as automatically larger since it 'looks positive'. Misconception: A mix of negative decimals, zero, and a fraction is ordered without converting to a common form first.

15. Order from least to greatest: $-\frac{3}{8}$, -0.35 , -0.4 .

15. -0.4 , $-\frac{3}{8}$, -0.35 . Likely wrong answer: orders by digit count instead of actual value. Misconception: Negative decimals and a fraction with different digit counts are ordered without converting to a common form first.

16. Calculate: $\frac{3}{8} + 0.25$

16. $\frac{5}{8} = 0.625$. Likely wrong answer: adds without converting the fraction and decimal to the same form first. Misconception: A fraction and a decimal are added without first converting one to match the other's form.

17. Calculate: $0.6 - \frac{1}{4}$

17. $7/20 = 0.35$. Likely wrong answer: subtracts without converting the fraction and decimal to the same form first. Misconception: A fraction and a decimal are subtracted without first converting one to match the other's form.

18. Calculate: $-1/4 \times 8$

18. $-2 = -2$. Likely wrong answer: $-2/4$ left unsimplified, or the sign dropped. Misconception: The product is left unsimplified, or the negative sign is dropped during multiplication.

Worksheet C: Mixed Application

1. A student says $-0.3 = -1/3$. Explain the mistake.

1. $-0.3 = -3/10$, while $-1/3 = -0.333\dots$; they are not equal. Likely wrong answer: accepts $-1/3$. Misconception: The digit 3 is used as a denominator without considering place value. Next move: Read 0.3 as three tenths before writing the fraction.

2. Is 0.1234567891011... (digits keep counting up, never repeating) rational?

2. No. It never terminates and never repeats, so it cannot be written as a fraction. Likely wrong answer: Yes (assumes any decimal, however it behaves, must be rational). Misconception: A non-repeating, non-terminating decimal is not recognized as excluded from the rational numbers.

3. Compare: $-5/6$ ____ -0.8

3. -0.8 ($-0.83333333333333333333333333333333$ vs -0.8). Likely wrong answer: $-5/6$ (compares negative values as if the larger digit means the greater value). Misconception: A bigger digit on a negative decimal or fraction is mistaken for a bigger value.

4. Write 0.36 as a fraction, then convert your answer back to a decimal to check.

4. $0.36 = 36/100 = 9/25$, and $9 \div 25 = 0.36$. Likely wrong answer: $36/10$. Misconception: Hundredths are treated as tenths or the fraction is not simplified. Next move: Name the decimal place, write the power-of-ten denominator, then use the GCF.

5. Compare: 0.45 ____ $4/9$

5. 0.45 ($0.44444444444444444444444444444444$ vs 0.45). Likely wrong answer: $4/9$ (compares without converting to matching forms first). Misconception: A fraction and a decimal are compared by appearance instead of converting to the same form.

6. A sprinter's time is 9.58 s. Write it as a fraction over 100.

6. $958/100 = 479/50$ simplified. Likely wrong answer: $958/1000$ left unsimplified, or the wrong denominator used. Misconception: A decimal is written as a fraction using the wrong power of ten for the denominator, or left unsimplified.

7. Order from least to greatest: $-7/8$, -0.9 , 0 , 0.6

7. -0.9 , $-7/8$, 0 , 0.6 . Likely wrong answer: orders by treating the fraction as automatically the most negative. Misconception: Negative fractions and decimals are ordered without converting to a common form first.

8. Compare: $-3/4$ ____ -0.7

8. -0.7 (-0.75 vs -0.7). Likely wrong answer: $-3/4$ (compares negative values as if the larger digit means the greater value). Misconception: A bigger digit on a negative decimal or fraction is mistaken for a bigger value.

9. Write $5/6$ as a decimal.

9. $0.8333...$ (repeating). Likely wrong answer: 0.83 (rounds the repeating decimal incorrectly or stops it too early). Misconception: A repeating decimal is truncated in a way that changes its value instead of correctly showing the repeat.

10. A recipe calls for $2/3$ cup of sugar. A digital scale reads in decimals. What decimal should be entered?

10. $0.6667...$ (repeating). Likely wrong answer: 0.66 or 0.67 (rounds the repeating decimal in a way that changes the intended value). Misconception: A repeating decimal is truncated or rounded without preserving enough precision for the context.

11. Give a rational number between 0.5 and $3/5$.

11. Example: $0.55 = 11/20$, which lies between 0.5 and $3/5 = 0.6$. Likely wrong answer: gives 0.5 or 0.6 . Misconception: An endpoint is offered instead of a value strictly between. Next move: Convert both endpoints to hundredths and choose an interior value.

12. Explain why $1/6$ is rational even though its decimal form never terminates.

12. $1/6 = 0.1667...$, a repeating decimal. Repeating decimals are rational because they can be written as a fraction of two integers. Likely wrong answer: says it isn't rational

because the decimal never ends. Misconception: A repeating decimal is confused with a non-repeating one when applying the definition of rational.

13. Compare: -0.125 ____ $-1/8$

13. Equal: $-0.125 = -1/8$. Likely wrong answer: says they're different because one is written as a fraction and one isn't. Misconception: Two forms of the same value are judged unequal based on appearance instead of actual value.

14. A temperature is -3.5°C . Write it as a mixed-sign fraction.

14. $-7/2$. Likely wrong answer: $7/2$ without the negative sign carried through. Misconception: The negative sign from the original decimal is dropped when converting to a fraction.

15. Explain why $1/0$ is not a rational number.

15. Division by zero is undefined. There is no value that $1/0$ could equal, so it fails the definition of a rational number entirely. Likely wrong answer: says it's because the answer would be negative. Misconception: The explanation focuses on sign rather than the actual reason division by zero is undefined.

Answer Key — Exit Ticket

1. Yes ($-6 = \frac{-6}{1}$).
2. $\frac{11}{25}$.
3. $\frac{5}{8}$ ($0.625 > 0.6$).