

Chapter 6 — Integer Operations

Teacher Chapter-at-a-Glance

Materials

A number line, printed or drawn on the board. No other materials required.

Talk Prompt

Ask the class to calculate $7 - (-5)$ before teaching the rewrite rule. Many will drop the negative sign and answer 2 instead of 12. Use that guess to introduce why the rewrite step is not optional.

If Ready-to-Learn Reveals a Gap

If two or more Ready-to-Learn items reveal a gap, stop and rebuild integer sense (comparing negatives, opposites, absolute value) before moving into operations. Every rule here depends on that foundation.

Mastery Decision Guide

What a missed Tripwire item actually means, and what to do about it before moving on.

Missed Tripwire #1 → Forgetting to add the opposite when subtracting (Mistake 2). A

subtraction answer that matches what you'd get from just dropping the negative sign, rather than flipping it, signals this error. Review before the next chapter.

Missed Tripwire #2 → Using "two negatives make a positive" for addition (Mistake 1). A

positive answer to an addition problem where both addends were negative signals this exact error. Review before the next chapter.

Ready-to-Learn Check

Quick check before starting. A wrong answer below points to a specific gap worth closing first, not just a miss to shrug off.

1. Which is greater, -8 or -3 ?

Correct answer: -3

Likely wrong answer: -8

Misconception: Assumes a bigger digit means a bigger negative number.

Next move: Have the student picture both on a number line before answering.

2. What is the opposite of -6 ?

Correct answer: 6

Likely wrong answer: -6

Misconception: Forgets to flip the sign, or flips it twice back to the original.

Next move: Have the student picture the number's mirror image across zero.

3. Evaluate $|-9|$.

Correct answer: 9

Likely wrong answer: -9

Misconception: Writes absolute value as negative, keeping the original sign.

Next move: Have the student count the spaces from the number to zero, which is never a negative count.

Common Mistakes

Students often run into the same kinds of trouble with this topic. Recognizing these patterns helps avoid them.

Comparing 0.86 and 0.809 by Place Value

	Ones		Tenths	Hundredths	Thousandths
0.86	0	.	8	6	0
0.809	0	.	8	0	9

Ones tie. Tenths tie. Hundredths decide it: $6 > 0$, so $0.86 > 0.809$.

Same signs give a positive result. Different signs give a negative result. For $\times$ and $\div$ only.

Mistake 1: Using "two negatives make a positive" for addition

What students do: Calculating $-6 + (-2)$ and expecting a positive answer because both numbers are negative.

Why this happens: The phrase "two negatives make a positive" is true and useful, but only for multiplication and division. It gets misapplied to addition because it's easy to remember and feels like it should be universal.

What to do instead: For addition, same signs mean ADD the sizes and KEEP that sign. Two negatives added together give a bigger negative, not a positive.

Diagnostic trigger: A positive answer to an addition problem where both addends were negative signals this exact error.

Mistake 2: Forgetting to add the opposite when subtracting

What students do: Calculating $7 - (-5)$ by dropping the negative sign and just subtracting the size, instead of rewriting the expression first.

Why this happens: The rewrite step feels like an extra step that can be skipped when the answer seems obvious.

What to do instead: Every subtraction of an integer gets rewritten as addition of the opposite first, with no exceptions, even when it looks simple.

Diagnostic trigger: A subtraction answer that matches what you'd get from just dropping the negative sign, rather than flipping it, signals this error.

Mistake 3: Adding different signs incorrectly

What students do: Adding the sizes of -9 and 4 instead of subtracting them.

Why this happens: Addition usually means combining sizes, so it's easy to add sizes out of habit even when the signs are different.

What to do instead: Different signs mean the numbers partly cancel. Subtract the smaller size from the larger, and keep the sign of the larger number.

Diagnostic trigger: An answer whose size is the SUM of the two original sizes, when the signs were different, signals this error.

Mistake 4: Dropping brackets too quickly

What students do: Rewriting $-6 - (-9)$ as $-6 + 9$ without checking that the sign flip was applied correctly.

Why this happens: The rewrite step involves two sign changes at once, the operation and the second number, and it's easy to only catch one of them.

What to do instead: Rewrite one piece at a time: change the operation from subtraction to addition, then separately flip the sign of the second number. Check both before moving on.

***Diagnostic trigger:** A rewritten expression with the same sign on the second number as the original signals this error.*

Mistake 5: Attempting division by zero

What students do: Trying to calculate an expression like $-12 \div 0$.

Why this happens: Zero looks like an ordinary number, so it's easy to assume every division involving it works like any other division.

What to do instead: Division by zero is undefined. There is no number that makes the division true, so this expression simply has no answer.

***Diagnostic trigger:** Any answer at all given for a division problem with a zero divisor signals this error, since the correct response is that it's undefined.*

Practice

Teacher solutions follow this practice set.

1. Calculate: $-4 + (-6)$
2. Calculate: $-9 + 5$
3. Calculate: $8 + (-3)$
4. Calculate: $6 - (-9)$
5. Calculate: $-5 - 8$
6. Calculate: $-7 - (-12)$
7. Calculate: -6×8
8. Calculate: $-7 \times (-9)$
9. Calculate: $-42 \div 6$
10. Calculate: $-54 \div (-9)$
11. Calculate: $-6 + 15 - (-3)$
12. The temperature is -8°C and rises 12°C . What is the new temperature?
13. A balance is -40 and $\$25$ is deposited. What is the new balance?

Practice Solutions

Use these concise solutions to check the method as well as the final answer.

1. Calculate: $-4 + (-6)$

Solution: $-4 + (-6) = -10$; add the magnitudes and keep the negative sign.

2. Calculate: $-9 + 5$

Solution: $-9 + 5 = -4$; subtract 5 from 9 and keep the sign of the larger magnitude.

3. Calculate: $8 + (-3)$

Solution: $8 + (-3) = 8 - 3 = 5$.

4. Calculate: $6 - (-9)$

Solution: $6 - (-9) = 6 + 9 = 15$.

5. Calculate: $-5 - 8$

Solution: $-5 - 8 = -5 + (-8) = -13$.

6. Calculate: $-7 - (-12)$

Solution: $-7 - (-12) = -7 + 12 = 5$.

7. Calculate: -6×8

Solution: $-6 \times 8 = -48$; factors with different signs give a negative product.

8. Calculate: $-7 \times (-9)$

Solution: $-7 \times (-9) = 63$; two negative factors give a positive product.

9. Calculate: $-42 \div 6$

Solution: $-42 \div 6 = -7$; different signs give a negative quotient.

10. Calculate: $-54 \div (-9)$

Solution: $-54 \div (-9) = 6$; two negative values give a positive quotient.

11. Calculate: $-6 + 15 - (-3)$

Solution: $-6 + 15 - (-3) = -6 + 15 + 3 = 12$.

12. The temperature is -8°C and rises 12°C . What is the new temperature?

Solution: $-8^{\circ}\text{C} + 12^{\circ}\text{C} = 4^{\circ}\text{C}$.

13. A balance is -40 and $\$25$ is deposited. What is the new balance?

Solution: $-\$40 + \$25 = -\$15$, so the new balance is $-\$15$.

Tripwire Check

These questions test whether the thinking is solid — not just whether the procedure was memorized. Each wrong answer below points to a specific misconception, not just "try again."

1. Calculate: $9 - (-6)$

Correct answer: 15. Rewritten as $9 + 6$.

Likely wrong answer: 3, from dropping the negative sign instead of flipping it.

Misconception: Subtracts the size of the second number directly, without rewriting the subtraction as addition of the opposite first.

Next move: Have the student say the rewrite out loud before calculating anything: "subtracting a negative becomes adding a positive."

2. Calculate: $-8 + (-3)$

Correct answer: -11. Same signs added get bigger, not smaller or positive.

Likely wrong answer: 24, from applying the multiplication sign rule to an addition problem.

Misconception: Applies "two negatives make a positive" to addition, when that rule belongs to multiplication and division only.

Next move: Have the student name which operation is in front of them before picking a sign rule: addition and subtraction use one rule, multiplication and division use a completely different one.

Answer Key — Worksheets (Full Diagnostic)

Worksheet A: Guided Practice

1. Calculate: $-2 + 7$

1. 5. Likely wrong answer: -9 (adds the sizes and keeps a negative sign). Misconception: Different-sign addition is treated as same-sign addition, adding the sizes instead of subtracting them.

2. Calculate: $6 - 2$

2. 4. Likely wrong answer: 8 (adds instead of subtracting two positives). Misconception: A basic subtraction between two positive numbers is treated as addition.

3. Calculate: -3×5

3. -15. Likely wrong answer: 15 (drops the sign rule for multiplying different signs). Misconception: A negative and a positive multiplied together are treated as if same-signed.

4. Calculate: $-9 + 4$

4. -5. Likely wrong answer: 5 or 13 (mishandles the sign when combining different-signed numbers). Misconception: The larger size's sign is not correctly carried into the result.

5. Calculate: $6 - (-2)$

5. 8. Likely wrong answer: 4 (subtracts the negative instead of rewriting it as addition). Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

6. Calculate: $18 \div (-3)$

6. -6. Likely wrong answer: 6 (drops the sign rule for dividing different signs). Misconception: A negative and a positive divided together are treated as if same-signed.

7. Calculate: $-6 + (-9)$

7. -15. Likely wrong answer: -3 or 3 (subtracts the sizes instead of adding them for same-sign numbers). Misconception: Same-sign addition is treated as different-sign addition, subtracting the sizes instead of adding them.

8. Calculate: $-4 - 5$

8. -9. Likely wrong answer: -1 (treats subtracting a positive as if it flips to addition). Misconception: Subtracting a positive number is mistakenly rewritten as addition.

9. Calculate: $-3 \times (-5)$

9. 15. Likely wrong answer: -15 (drops the sign rule for multiplying same signs). Misconception: Two negatives multiplied together are treated as staying negative.

10. Calculate: $9 + (-5)$

10. 4. Likely wrong answer: -4 or 14 (mishandles the sign when combining different-signed numbers). Misconception: The larger size's sign is not correctly carried into the result.

11. Calculate: $-6 - (-5)$

11. -1. Likely wrong answer: -11 (subtracts the negative instead of rewriting it as addition). Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

12. Calculate: $-18 \div (-3)$

12. 6. Likely wrong answer: -6 (drops the sign rule for dividing same signs).

Misconception: Two negatives divided together are treated as staying negative.

13. Calculate: $-2 - (-5)$

13. 3. Likely wrong answer: -7 (subtracts the negative instead of rewriting it as addition).

Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

14. Calculate: $8 - (-3)$

14. 11. Likely wrong answer: 5 (subtracts the negative instead of rewriting it as addition).

Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

15. Calculate: $-7 + 5$

15. -2 . Likely wrong answer: 2 or 12 (mishandles the sign when combining different-signed numbers). Misconception: The larger size's sign is not correctly carried into the result.

Worksheet B: Independent Practice

1. Calculate: $6 + (-12)$

1. -6 . Likely wrong answer: 18 or -18 (mishandles the sign when combining different-signed numbers). Misconception: The larger size's sign is not correctly carried into the result.

2. Calculate: $4 - 10$

2. -6 . Likely wrong answer: 6 (subtracts in the wrong direction, ignoring which number is larger). Misconception: The subtraction is performed as size minus size without tracking which value is actually being taken from which.

3. Calculate: -3×7

3. -21 . Likely wrong answer: 21 (drops the sign rule for multiplying different signs). Misconception: A negative and a positive multiplied together are treated as if same-signed.

4. Calculate: $30 \div (-5)$

4. -6. Likely wrong answer: 6 (drops the sign rule for dividing different signs).

Misconception: A negative and a positive divided together are treated as if same-signed.

5. Calculate: $-3 + (-5)$

5. -8. Likely wrong answer: 2 or -2 (subtracts the sizes instead of adding them for same-sign numbers). Misconception: Same-sign addition is treated as different-sign addition, subtracting the sizes instead of adding them.

6. Calculate: $4 - (-2)$

6. 6. Likely wrong answer: 2 (subtracts the negative instead of rewriting it as addition).

Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

7. Calculate: $8 \times (-4)$

7. -32. Likely wrong answer: 32 (drops the sign rule for multiplying different signs).

Misconception: A negative and a positive multiplied together are treated as if same-signed.

8. Calculate: $-48 \div 6$

8. -8. Likely wrong answer: 8 (drops the sign rule for dividing different signs).

Misconception: A negative and a positive divided together are treated as if same-signed.

9. Calculate: $-8 + (-9)$

9. -17. Likely wrong answer: -1 or 1 (subtracts the sizes instead of adding them for same-sign numbers). Misconception: Same-sign addition is treated as different-sign addition, subtracting the sizes instead of adding them.

10. Calculate: $-7 - (-13)$

10. 6. Likely wrong answer: -20 (subtracts the negative instead of rewriting it as addition). Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

11. Calculate: $-5 \times (-9)$

11. 45. Likely wrong answer: -45 (drops the sign rule for multiplying same signs).

Misconception: Two negatives multiplied together are treated as staying negative.

12. Calculate: $-56 \div (-7)$

12. 8. Likely wrong answer: -8 (drops the sign rule for dividing same signs).

Misconception: Two negatives divided together are treated as staying negative.

13. Calculate: $-8 - 8$

13. -16 . Likely wrong answer: 0 or 16 (mishandles combining two negative numbers).

Misconception: Two negative numbers subtracted are combined as if their signs cancel out.

14. Calculate: $-9 - (-5)$

14. -4 . Likely wrong answer: -14 (subtracts the negative instead of rewriting it as addition). Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

15. Calculate: $0 \times (-13)$

15. 0 . Likely wrong answer: -13 (assumes multiplying by zero still carries the other factor's sign into a nonzero result). Misconception: Multiplying by zero is not recognized as always giving zero regardless of the other factor's sign.

16. Calculate: $-4 + 7 - (-5)$

16. 8 . Likely wrong answer: mishandles one of the two sign changes in the multi-step expression. Misconception: One of several sign changes in a multi-step expression is applied incorrectly while the others are handled correctly.

17. Calculate: $-6 - (-2) + 9$

17. 5 . Likely wrong answer: mishandles one of the two sign changes in the multi-step expression. Misconception: One of several sign changes in a multi-step expression is applied incorrectly while the others are handled correctly.

18. Calculate: $-3 \times 5 + 4$

18. -11 . Likely wrong answer: computes the addition before the multiplication. Misconception: Multiplication is not completed before addition in a multi-step expression.

Worksheet C: Mixed Application

1. A student says $-8 + 3 = 11$. Explain the mistake.

1. $-8 + 3 = -5$. Likely wrong answer: 11. Misconception: Absolute values are added and the sign is ignored. Next move: Start at -8 on a number line and move 3 units right.

2. The temperature is -9°C and rises 14°C . New temperature?

2. 5°C . Likely wrong answer: -23°C (subtracts the rise instead of adding it).

Misconception: A temperature increase is modeled with subtraction instead of addition.

3. Calculate: $5 - (-4)$

3. 9. Likely wrong answer: 1 (subtracts the negative instead of rewriting it as addition).

Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

4. A diver is at -4 m and goes down 7 m. Where is the diver?

4. -11 m. Likely wrong answer: 3 m (adds instead of subtracting further depth).

Misconception: Going further down from a negative depth is modeled with addition instead of subtraction.

5. Explain why $-6 + (-2)$ is not positive.

5. $-6 + (-2) = -8$, not a positive number. Likely wrong answer: 8. Misconception: The multiplication sign rule is incorrectly applied to addition. Next move: Name the operation, then combine two negative movements on a number line.

6. A bank account is -42 and $\$30$ is deposited. New balance?

6. -12 . Likely wrong answer: -72 (subtracts the deposit instead of adding it).

Misconception: A deposit into a negative balance is modeled with subtraction instead of addition.

7. Calculate: $-5 + 12 - (-5)$

7. 12. Likely wrong answer: mishandles the double sign change (subtracting a negative) within the multi-step expression. Misconception: One of several sign changes in a multi-step expression is applied incorrectly while the others are handled correctly.

8. A team loses 6 yards on each of 4 plays. Total change?

8. -24 yards. Likely wrong answer: -6 yards (finds the loss on one play instead of multiplying by the number of plays). Misconception: A repeated equal loss across several plays is not multiplied by the number of times it happens.

9. Calculate: $6 - (-3) + 5$

9. 14. Likely wrong answer: mishandles the sign when subtracting the negative in the first step. Misconception: Subtracting a negative is treated as an ordinary subtraction instead of rewritten as adding the opposite.

10. A debt of -60 is split equally among 5 people. Each person's share?

10. -12 . Likely wrong answer: -300 (multiplies the debt by the number of people instead of dividing). Misconception: The wrong operation is chosen for splitting a total into equal shares.

11. Calculate: $-4 \times (-3) \div 2$

11. 6. Likely wrong answer: mishandles the sign of the product before dividing. Misconception: The sign rule for multiplying two negatives is applied incorrectly before the division step.

12. Explain why $-3 - (-7) = 4$.

12. Subtracting -7 means adding its opposite: $-3 - (-7) = -3 + 7 = 4$. Likely wrong answer: -10 . Misconception: The second negative is not rewritten as its opposite. Next move: Rewrite subtraction as addition before calculating.

13. Write an addition expression with a value of -4 .

13. Example: $1 + (-5) = -4$ (any pair that adds to -4 works). Likely wrong answer: gives an expression that doesn't actually equal -4 when checked. Misconception: An expression is proposed without checking that it truly evaluates to the required value.

14. Write a subtraction expression with a value of 9.

14. Example: $14 - 5 = 9$ (any pair that subtracts to 9 works). Likely wrong answer: gives an expression that doesn't actually equal 9 when checked. Misconception: An expression is proposed without checking that it truly evaluates to the required value.

15. Compare -4×5 and $-4 + 5$. Which is greater?

15. $-4 + 5 = 1$, which is greater. Likely wrong answer: says -4×5 is greater without evaluating both expressions. Misconception: A comparison is made by appearance rather than by actually calculating both values.

Answer Key — Exit Ticket

1. -14.
2. 12.
3. 24.