

Chapter 4 — Decimal Sense and Decimal Operations

Teacher Chapter-at-a-Glance

Materials

A place-value chart, printed or drawn on the board. No other materials required.

Talk Prompt

Ask the class to estimate 3.8×5.2 before calculating it exactly. Most will land near 20. Use that shared estimate as the check once the exact answer comes out.

If Ready-to-Learn Reveals a Gap

If two or more Ready-to-Learn items reveal a gap, stop and rebuild whole-number operations and decimal comparison before moving into decimal operations. All four operations here depend on both.

Mastery Decision Guide

What a missed Tripwire item actually means, and what to do about it before moving on.

Missed Tripwire #1 → Lining up decimal points to multiply (Mistake 1). *A multiplication set up with the decimal points visually aligned, the way an addition problem would be stacked, signals this error.* Review before the next chapter.

Missed Tripwire #2 → Shifting only one number when dividing by a decimal (Mistake 3). *A quotient that is off by a factor of 10 from a reasonable estimate signals this error.* Review before the next chapter.

Ready-to-Learn Check

Quick check before starting. A wrong answer below points to a specific gap worth closing first, not just a miss to shrug off.

1. What is $47 + 28$?

Correct answer: 75

Likely wrong answer: 65

Misconception: Loses track of a regrouped ten when adding the tens digits.

Next move: Have the student add the ones first, write down any regrouped ten, then add the tens including it.

2. What is 6×7 ?

Correct answer: 42

Likely wrong answer: 35

Misconception: Confuses a nearby multiplication fact.

Next move: Have the student build the fact from repeated addition.

3. Compare: 0.6 ____ 0.58

Correct answer: $0.6 > 0.58$

Likely wrong answer: $0.6 < 0.58$

Misconception: Assumes the decimal with more digits after the point is bigger.

Next move: Have the student line up both numbers at the decimal point before comparing any digit.

Common Mistakes

Students often run into the same kinds of trouble with this topic. Recognizing these patterns helps avoid them.

Comparing 0.86 and 0.809 by Place Value

	Ones		Tenths	Hundredths	Thousandths
0.86	0	.	8	6	0
0.809	0	.	8	0	9

Ones tie. Tenths tie. Hundredths decide it: $6 > 0$, so $0.86 > 0.809$.

$8.4 \div 0.7$ and $84 \div 7$ are the same division. Both numbers shifted together.

Mistake 1: Lining up decimal points to multiply

What students do: Writing two decimals lined up at the decimal point before multiplying, the way addition and subtraction are set up.

Why this happens: Lining up decimal points is the correct first move for two of the four operations, so it feels like a safe habit to repeat for all of them.

What to do instead: Ignore the decimal points during the multiplication itself. Multiply the digits as whole numbers, then place the decimal by counting total decimal places across both factors.

Diagnostic trigger: A multiplication set up with the decimal points visually aligned, the way an addition problem would be stacked, signals this error.

Mistake 2: Forgetting a placeholder zero when subtracting

What students do: Subtracting $5 - 0.46$ without first writing the whole number with matching decimal places.

Why this happens: A whole number does not show a decimal point at all, so it is easy to forget it needs one, with zeros, before lining anything up.

What to do instead: Give every whole number a decimal point and the same number of decimal places as the other number before subtracting.

Diagnostic trigger: A subtraction answer with too few decimal places, or a borrowing error at the decimal point, signals this error.

Mistake 3: Shifting only one number when dividing by a decimal

What students do: Multiplying the divisor by a power of ten to make it whole, but leaving the dividend unchanged.

Why this happens: The divisor is the number that visibly needs fixing, so it gets all the attention while the dividend gets skipped.

What to do instead: Whatever power of ten clears the divisor's decimal point must multiply the dividend too. Both numbers move together, every time.

Diagnostic trigger: A quotient that is off by a factor of 10 from a reasonable estimate signals this error.

Mistake 4: Skipping estimation

What students do: Calculating first and never checking the answer against a rough estimate.

Why this happens: Estimating feels like an extra step when the calculation itself seems straightforward.

What to do instead: Round each number to a friendly value and solve that easier problem first, before the real calculation. Compare the two.

Diagnostic trigger: An answer that is off by a factor of 10 or 100, submitted with no estimate shown anywhere, signals this habit is missing.

Mistake 5: Assuming division always makes a number smaller

What students do: Expecting $4.5 \div 0.5$ to be smaller than 4.5.

Why this happens: Division usually shrinks a number when dividing by something bigger than 1, so it feels like a rule that should always hold.

What to do instead: Dividing by a number less than 1 makes the result bigger, not smaller. It is asking how many small pieces fit into the original amount.

***Diagnostic trigger:** An answer smaller than the dividend, when the divisor is less than 1, signals this error.*

Practice

Teacher solutions follow this practice set.

1. Add: $4.25 + 3.6$
2. Subtract: $8.6 - 2.75$
3. Multiply: 1.5×4
4. Estimate, then multiply: 3.2×2.4
5. Divide: $9.6 \div 4$
6. Divide: $7.2 \div 0.8$
7. A jug holds 1.375 L of juice. Another holds 0.85 L. How much total?
8. A ribbon is 6.5 m. You cut off 3.25 m. How much remains?
9. A cyclist rides 2.1 km each day for 2 days. How far in total?
10. Explain why $4.5 \div 0.5$ is greater than 4.5.

Practice Solutions

Use these concise solutions to check the method as well as the final answer.

1. Add: $4.25 + 3.6$
Solution: Align decimal points: $4.25 + 3.60 = 7.85$.
2. Subtract: $8.6 - 2.75$
Solution: Align decimal points: $8.60 - 2.75 = 5.85$.
3. Multiply: 1.5×4
Solution: $1.5 \times 4 = 6$.

4. Estimate, then multiply: 3.2×2.4

Solution: Estimate: $3 \times 2 = 6$. Exact calculation: $32 \times 24 = 768$, then place two decimal digits, giving 7.68.

5. Divide: $9.6 \div 4$

Solution: $9.6 \div 4 = 2.4$.

6. Divide: $7.2 \div 0.8$

Solution: Multiply both numbers by 10: $7.2 \div 0.8 = 72 \div 8 = 9$.

7. A jug holds 1.375 L of juice. Another holds 0.85 L. How much total?

Solution: $1.375 \text{ L} + 0.850 \text{ L} = 2.225 \text{ L}$.

8. A ribbon is 6.5 m. You cut off 3.25 m. How much remains?

Solution: $6.50 \text{ m} - 3.25 \text{ m} = 3.25 \text{ m}$.

9. A cyclist rides 2.1 km each day for 2 days. How far in total?

Solution: $2.1 \text{ km} \times 2 = 4.2 \text{ km}$.

10. Explain why $4.5 \div 0.5$ is greater than 4.5.

Solution: $4.5 \div 0.5 = 45 \div 5 = 9$. Dividing by one-half counts how many halves fit into 4.5, so the quotient is greater than 4.5.

Tripwire Check

These questions test whether the thinking is solid, not just whether the procedure was memorized. Each wrong answer below points to a specific misconception, not just "try again."

1. Multiply: 3.5×1.6

Correct answer: 5.6 (2 total decimal places from both factors).

Likely wrong answer: 56, from counting only one factor's decimal place.

Misconception: Counts decimal places from only one factor instead of adding both factors' decimal places together.

Next move: Have the student label each factor's decimal-place count separately, then add the two labels before placing the decimal in the answer.

2. Divide: $6.3 \div 0.9$

Correct answer: 7 (both numbers shifted by $\times 10$).

Likely wrong answer: 0.7, from shifting only the divisor.

Misconception: Multiplies the divisor by a power of ten to clear its decimal point, but leaves the dividend unshifted.

Next move: Have the student write both numbers side by side and cross out one zero on each at the same time, so the shift is visibly matched.

Answer Key — Worksheets (Full Diagnostic)

Worksheet A: Guided Practice

1. Write 0.6 with two decimal places.

1. 0.60. Likely wrong answer: 0.6 left as is, or 0.06 (adds the zero in the wrong place).

Misconception: A trailing zero is added before the existing digit instead of after it.

2. Add: $2.4 + 1.75$

2. 4.15. Likely wrong answer: 2.79 (lines up the digits on the right instead of the decimal points). Misconception: The numbers are stacked by matching digit count instead of matching place value.

3. Subtract: $7 - 4.35$

3. 2.65. Likely wrong answer: 3.35 (subtracts without giving 7 a decimal point and matching zeros first). Misconception: A whole number is not given matching decimal places before subtracting.

4. Multiply: 1.4×3

4. 4.2. Likely wrong answer: 0.42 (misplaces the decimal point by counting the wrong number of places). Misconception: The total number of decimal places in the factors is miscounted when placing the decimal in the product.

5. Write 0.750 without the unnecessary final zero.

5. 0.75. Likely wrong answer: 0.075 (drops a digit instead of just the trailing zero).

Misconception: A digit that holds a real place value is removed along with the harmless trailing zero.

6. Divide: $6.4 \div 2$

6. 3.2. Likely wrong answer: 32 (loses track of the decimal point entirely). Misconception: The decimal point is not carried straight up into the quotient during division.

7. Add: $5.38 + 2.4$

7. 7.78. Likely wrong answer: 5.78 (adds 2.4 as if it had two decimal places already lined up on the right). Misconception: The shorter decimal is not given a matching zero before lining up the addition.

8. Multiply: 0.7×2

8. 1.4. Likely wrong answer: 0.14 (misplaces the decimal point in the product). Misconception: The total number of decimal places in the factors is miscounted when placing the decimal in the product.

9. Subtract: $5 - 0.46$

9. 4.54. Likely wrong answer: 4.44 (subtracts without giving 0.46 the same number of decimal places as 5.00). Misconception: The two numbers are subtracted without first matching their number of decimal places.

10. Divide: $4.5 \div 3$

10. 1.5. Likely wrong answer: 15 (loses track of the decimal point entirely). Misconception: The decimal point is not carried straight up into the quotient during division.

11. Multiply: 1.4×2.1

11. 2.94. Likely wrong answer: 29.4 or 0.294 (misplaces the decimal point in the product). Misconception: The total number of decimal places in the factors is miscounted when placing the decimal in the product.

12. Divide: $3.6 \div 0.4$

12. 9. Likely wrong answer: 0.9 (shifts only the dividend, not the divisor, before dividing). Misconception: Only one of the two numbers is adjusted to clear the decimal point before dividing.

13. Divide: $4.9 \div 0.7$

13. 7. Likely wrong answer: 0.7 (shifts only the dividend, not the divisor, before dividing). Misconception: Only one of the two numbers is adjusted to clear the decimal point before dividing.

14. Estimate: 2.8×4.9

14. About $3 \times 5 = 15$. Likely wrong answer: about $2 \times 5 = 10$ (rounds one factor in a way that isn't a friendly estimate). Misconception: A factor is rounded to a value that doesn't actually make the estimate easier to compute.

15. Explain why $0.3 \times 10 = 3$.

15. Multiplying by 10 shifts every digit one place value to the left, turning 0.3 into 3. Likely wrong answer: says the digits just move without explaining why. Misconception: The explanation states what happens without connecting it to place value shifting.

Worksheet B: Independent Practice

1. Write 0.800 without the unnecessary final zero.

1. 0.8. Likely wrong answer: 0.80 left with the zero, or 0.8 with a digit dropped incorrectly. Misconception: A trailing zero is treated as though removing it changes the value, or a real digit is removed along with it.

2. Add: $4.45 + 2.88$

2. 7.33. Likely wrong answer: 6.33 or 7.13 (misaligns the decimal points when adding). Misconception: The numbers are stacked by matching digit count instead of matching place value.

3. Subtract: $7.1 - 2.86$

3. 4.24. Likely wrong answer: 4.86 or 5.24 (subtracts without matching decimal places first). Misconception: The two numbers are subtracted without first matching their number of decimal places.

4. Multiply: 3.3×4

4. 13.2. Likely wrong answer: 12.2 or 7.2 (misplaces the decimal point in the product). Misconception: The total number of decimal places in the factor is miscounted when placing the decimal in the product.

5. Compare: 0.56 ____ 0.506

5. $0.56 > 0.506$. Likely wrong answer: $0.56 < 0.506$ (compares digit count instead of place value). Misconception: More digits after the decimal point is assumed to mean a larger value.

6. Divide: $8.4 \div 3$

6. 2.8. Likely wrong answer: 0.28 or 28 (loses track of the decimal point entirely).

Misconception: The decimal point is not carried straight up into the quotient during division.

7. Add: $8.09 + 5.4$

7. 13.49. Likely wrong answer: 8.13 or 13.44 (misaligns the decimal points when adding).

Misconception: The shorter decimal is not given a matching zero before lining up the addition.

8. Multiply: 0.6×3

8. 1.8. Likely wrong answer: 0.18 (misplaces the decimal point in the product).

Misconception: The total number of decimal places in the factor is miscounted when placing the decimal in the product.

9. Subtract: $5 - 0.66$

9. 4.34. Likely wrong answer: 4.44 (subtracts without matching decimal places first).

Misconception: The two numbers are subtracted without first matching their number of decimal places.

10. Divide: $6.5 \div 5$

10. 1.3. Likely wrong answer: 13 (loses track of the decimal point entirely).

Misconception: The decimal point is not carried straight up into the quotient during division.

11. Add: $3.75 + 4.7$

11. 8.45. Likely wrong answer: 8.15 or 7.45 (misaligns the decimal points when adding).

Misconception: The shorter decimal is not given a matching zero before lining up the addition.

12. Multiply: 5.2×1.3

12. 6.76. Likely wrong answer: 6.76 with the decimal misplaced (e.g. 67.6 or 0.676).

Misconception: The total number of decimal places in the factors is miscounted when placing the decimal in the product.

13. Subtract: $16.25 - 3.862$

13. 12.388. Likely wrong answer: 12.388 with a digit-count error from misaligned subtraction. Misconception: Numbers with different decimal-place counts are subtracted without matching their places first.

14. Divide: $7.2 \div 0.9$

14. 8. Likely wrong answer: 0.8 (shifts only the dividend, not the divisor, before dividing). Misconception: Only one of the two numbers is adjusted to clear the decimal point before dividing.

15. Multiply: 0.4×0.6

15. 0.24. Likely wrong answer: 2.4 (misplaces the decimal point in the product). Misconception: The total number of decimal places in the factors is miscounted when placing the decimal in the product.

16. Divide: $4.8 \div 0.6$

16. 8. Likely wrong answer: 0.8 (shifts only the dividend, not the divisor, before dividing). Misconception: Only one of the two numbers is adjusted to clear the decimal point before dividing.

17. Estimate, then multiply: 0.62×4.8

17. Estimate: $1 \times 5 = 5$. Exact: 2.976. Likely wrong answer: reasonable estimate but a miscalculated exact answer with the decimal misplaced. Misconception: The estimate is found correctly, but the exact product's decimal placement is miscounted separately.

18. Estimate, then multiply: 2.9×3.1

18. Estimate: $3 \times 3 = 9$. Exact: 8.99. Likely wrong answer: reasonable estimate but a miscalculated exact answer with the decimal misplaced. Misconception: The estimate is found correctly, but the exact product's decimal placement is miscounted separately.

Worksheet C: Mixed Application

1. A student says $0.6 + 0.25 = 0.31$. Explain the mistake and give the correct answer.

1. $0.6 + 0.25 = 0.60 + 0.25 = 0.85$. Likely wrong answer: 0.31. Misconception: Digits are added without aligning place values. Next move: Write 0.6 as 0.60 and align decimal points.

2. A student calculates $3.8 \times 5.2 = 197.6$. Explain the mistake and correct it.

2. $3.8 \times 5.2 = 19.76$. Likely wrong answer: 197.6. Misconception: The decimal is placed without counting the two total decimal places or estimating magnitude. Next move: Estimate $4 \times 5 \approx 20$, then place the decimal.

3. Two swimmers finish in 11.28 s and 11.3 s. Who is faster?

3. The swimmer who finished in 11.28 s. Likely wrong answer: 11.3 s is faster (assumes the larger-looking number is faster). Misconception: A smaller time value is not recognized as representing less time, so the faster swimmer is misidentified.

4. Three bottles each hold 2 L. What is the total volume?

4. 6 L. Likely wrong answer: 2 L (multiplies incorrectly or adds only two of the three bottles). Misconception: Not all of the equal groups given in the problem are included in the total.

5. A rope is 8.25 m and 4.6 m is cut off. How much remains?

5. 3.65 m. Likely wrong answer: 4.35 m (subtracts without matching decimal places first). Misconception: The two numbers are subtracted without first matching their number of decimal places.

6. A student rewrites $6.4 \div 0.8$ as $6.4 \div 8$. Explain the mistake.

6. Multiply both numbers by 10: $6.4 \div 0.8 = 64 \div 8 = 8$. Likely wrong answer: 0.8 from changing only the divisor. Misconception: The quotient is changed because the dividend and divisor are not scaled together. Next move: Show the equivalent division statement with both numbers multiplied by 10.

7. A bag of rice holds 5.4 kg, split into 3 equal portions. How much is each portion?

7. 1.8 kg. Likely wrong answer: 1.8 kg found by subtracting instead of dividing. Misconception: The wrong operation is chosen for splitting a total into equal portions.

8. A price is \$6.20. You buy 3 items. What is the total?

8. \$18.60. Likely wrong answer: \$18.20 or \$18.6 with the decimal misplaced. Misconception: The total number of decimal places in the product is miscounted.

9. Explain why $4.8 \div 0.8$ gives an answer greater than 4.8.

9. 0.8 is less than 1, so the answer counts how many pieces smaller than one fit into 4.8. That count is 6, bigger than 4.8. Likely wrong answer: says it's because division always

makes numbers smaller, with no connection to the divisor's size. Misconception: The explanation doesn't connect the result to the divisor being less than 1.

10. A runner runs 4.625 km each day for 2 days. Total distance?

10. 9.25 km. Likely wrong answer: 9.25 miscalculated by adding without matching decimal places. Misconception: The two equal amounts are combined without lining up their decimal places first.

11. A recipe uses 1.75 cups of flour. Triple the recipe. How many cups?

11. 5.25 cups. Likely wrong answer: 5.25 miscalculated by adding 1.75 twice instead of three times. Misconception: Tripling a quantity is treated as doubling it.

12. A 6.3 m ribbon is cut into pieces of 0.7 m each. How many full pieces?

12. 9 full pieces. Likely wrong answer: 90 full pieces (shifts only the dividend before dividing). Misconception: Only one of the two numbers is adjusted to clear the decimal point before dividing.

13. Estimate, then multiply: 5.9×0.68

13. Estimate: $6 \times 1 = 6$. Exact: 4.012. Likely wrong answer: reasonable estimate but a miscalculated exact answer with the decimal misplaced. Misconception: The estimate is found correctly, but the exact product's decimal placement is miscounted separately.

14. Explain the difference between 0.4 and 0.04.

14. 0.4 is 10 times as large as 0.04. 0.4 is 4 tenths; 0.04 is 4 hundredths. Likely wrong answer: says they're the same value since both start with a 4. Misconception: The place value of the 4 in each number is not compared, so the tenfold difference is missed.

15. Create your own decimal multiplication problem with an answer close to 2.

15. Answers will vary. Example: $0.8 \times 2.5 = 2$. Likely wrong answer: gives a problem whose answer is not actually close to 2. Misconception: A problem is created without checking that its actual product matches the target value.

Answer Key — Exit Ticket

1. 6.55.
2. 3.22.
3. 9.