

Chapter 3 — Fraction Foundations

Teacher Chapter-at-a-Glance

Materials

Fraction strips or a printed fraction bar diagram, if available. Not required.

Talk Prompt

Ask the class which is bigger, $\frac{3}{5}$ or $\frac{2}{3}$, before teaching the method. Let the room split on it. The disagreement is the reason a method, not a guess, matters here.

If Ready-to-Learn Reveals a Gap

If two or more Ready-to-Learn items reveal a gap, stop and rebuild equal-groups sharing and basic multiplication fluency before moving into fraction meaning. Every idea in this chapter assumes those are solid.

Mastery Decision Guide

What a missed Tripwire item actually means, and what to do about it before moving on.

Missed Tripwire #1 → Assuming a larger denominator means a larger fraction (Mistake 1). *A comparison answer that picks the fraction with the larger denominator, with no common-denominator work shown, signals this exact error. Review before the next chapter.*

Missed Tripwire #2 → Stopping before a fraction is fully simplified (Mistake 3). *A simplified answer that still shares a common factor greater than 1 signals this error. Review before the next chapter.*

Ready-to-Learn Check

Quick check before starting. A wrong answer below points to a specific gap worth closing first, not just a miss to shrug off.

1. Share 12 candies equally among 4 friends. How many does each friend get?

Correct answer: 3

Likely wrong answer: 8

Misconception: Subtracts the number of friends instead of dividing, or loses track of what "equally" requires.

Next move: Have the student deal out one candy at a time to each friend in turn and count what each pile ends with.

2. What is 6×4 ?

Correct answer: 24

Likely wrong answer: 20

Misconception: Confuses a nearby multiplication fact.

Next move: Have the student build the fact from repeated addition: $4 + 4 + 4 + 4 + 4 + 4$.

3. Compare: 15 ____ 9

Correct answer: $15 > 9$

Likely wrong answer: $15 < 9$

Misconception: Reverses the direction of the inequality symbol.

Next move: Have the student say which number is bigger in words first, then choose the matching symbol.

Common Mistakes

Students often run into the same kinds of trouble with this topic. Recognizing these patterns helps avoid them.

Comparing 0.86 and 0.809 by Place Value

	Ones		Tenths	Hundredths	Thousandths
0.86	0	.	8	6	0
0.809	0	.	8	0	9

Ones tie. Tenths tie. Hundredths decide it: $6 > 0$, so $0.86 > 0.809$.

$\frac{2}{3}$ has fewer pieces than $\frac{3}{5}$, but more of the bar is shaded.

Mistake 1: Assuming a larger denominator means a larger fraction

What students do: Comparing $\frac{3}{5}$ and $\frac{2}{3}$ and picking $\frac{3}{5}$ as bigger because 5 is bigger than 3.

Why this happens: More pieces feels like it should mean more amount, the same way more items feels like more. But a bigger denominator means smaller pieces, and smaller pieces cost the comparison nothing on their own.

What to do instead: Rewrite both fractions with a common denominator, or compare against a benchmark like one-half, before deciding.

Diagnostic trigger: A comparison answer that picks the fraction with the larger denominator, with no common-denominator work shown, signals this exact error.

Mistake 2: Changing only one part when building an equivalent fraction

What students do: Multiplying only the denominator (or only the numerator) of $\frac{2}{3}$ to reach a target, instead of both.

Why this happens: The target denominator is the visible goal, so it's easy to focus only on reaching it and forget the numerator needs the identical change.

What to do instead: Whatever is done to the denominator must be done to the numerator, and vice versa. Multiply both by the same number, always.

Diagnostic trigger: An equivalent fraction where only one of the two parts changed by the expected factor signals this error.

Mistake 3: Stopping before a fraction is fully simplified

What students do: Simplifying $\frac{12}{18}$ by dividing by 2 to get $\frac{6}{9}$, then stopping, without noticing 3 still divides both.

Why this happens: Dividing by ANY common factor feels like simplifying, so the first common factor found often gets treated as the finish line.

What to do instead: Find the GCF specifically, not just any common factor, or check the result again for a common factor before calling it done.

Diagnostic trigger: A simplified answer that still shares a common factor greater than 1 signals this error.

Mistake 4: Treating improper fractions as wrong answers

What students do: Feeling like $\frac{7}{4}$ needs to be "fixed" into a mixed number before it counts as a real answer.

Why this happens: Fractions greater than 1 look unusual compared to the proper fractions introduced first, so they can feel like a mistake rather than a valid, different form.

What to do instead: An improper fraction and its matching mixed number are the exact same value, just written two different ways. Neither one is more correct than the other.

Diagnostic trigger: A correct improper fraction marked wrong, or automatically rewritten, when the question never asked for a mixed number, signals this error.

Mistake 5: Comparing fractions without a method

What students do: Guessing which fraction is bigger by eye instead of using a common denominator or a benchmark.

Why this happens: Two fractions that look close in size invite a guess, especially under time pressure.

What to do instead: Always compare using a method: a common denominator, matching numerators, or a benchmark like one-half. Guessing is the method that produces the larger-denominator trap.

Diagnostic trigger: A comparison answer with no shown method, right or wrong, signals this habit worth catching early.

Practice

Teacher solutions follow this practice set.

1. In $\frac{5}{9}$, identify the numerator and the denominator.

2. $\frac{3}{5} = \frac{?}{40}$

3. $\frac{4}{9} = \frac{?}{18}$

4. Simplify: $\frac{20}{30}$

5. Compare: $\frac{5}{6}$ and $\frac{7}{9}$

6. Order from least to greatest: $\frac{2}{5}, \frac{1}{3}, \frac{3}{4}$

7. Write $\frac{17}{5}$ as a mixed number.
8. Write $3\frac{2}{7}$ as an improper fraction.
9. Is $\frac{9}{9}$ equal to, less than, or greater than 1?
10. Explain why $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent.

Practice Solutions

Use these concise solutions to check the method as well as the final answer.

1. In $\frac{5}{9}$, identify the numerator and the denominator.

Solution: In $\frac{5}{9}$, the numerator is 5 and the denominator is 9.

2. $\frac{3}{5} = \frac{?}{40}$

Solution: Multiply numerator and denominator by 8: $\frac{3}{5} = \frac{24}{40}$.

3. $\frac{4}{9} = \frac{?}{18}$

Solution: Multiply numerator and denominator by 2: $\frac{4}{9} = \frac{8}{18}$.

4. Simplify: $\frac{20}{30}$

Solution: Divide numerator and denominator by 10: $\frac{20}{30} = \frac{2}{3}$.

5. Compare: $\frac{5}{6}$ and $\frac{7}{9}$

Solution: Use denominator 18: $\frac{5}{6} = \frac{15}{18}$ and $\frac{7}{9} = \frac{14}{18}$, so $\frac{5}{6} > \frac{7}{9}$.

6. Order from least to greatest: $\frac{2}{5}, \frac{1}{3}, \frac{3}{4}$

Solution: Using a common denominator of 60 gives $\frac{1}{3} = \frac{20}{60}$, $\frac{2}{5} = \frac{24}{60}$, and $\frac{3}{4} = \frac{45}{60}$. Therefore $\frac{1}{3} < \frac{2}{5} < \frac{3}{4}$.

7. Write $\frac{17}{5}$ as a mixed number.

Solution: $17 \div 5 = 3$ remainder 2, so $\frac{17}{5} = 3\frac{2}{5}$.

8. Write $3\frac{2}{7}$ as an improper fraction.

Solution: Multiply the whole number by the denominator and add the numerator: $(3 \times 7 + 2)/7 = \frac{23}{7}$.

9. Is $\frac{9}{9}$ equal to, less than, or greater than 1?

Solution: $9/9 = 1$ because the numerator and denominator are equal.

10. Explain why $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent.

Solution: Multiplying $3/4$ by $3/3$ gives $9/12$. Since $3/3 = 1$, the value does not change.

Tripwire Check

These questions test whether the thinking is solid — not just whether the procedure was memorized. Each wrong answer below points to a specific misconception, not just "try again."

1. Which is greater, $\frac{4}{7}$ or $\frac{3}{5}$?

Correct answer: $\frac{3}{5} \cdot \frac{4}{7} = \frac{20}{35}$ and $\frac{3}{5} = \frac{21}{35}$, and $21 > 20$.

Likely wrong answer: $\frac{4}{7}$, because 7 is the bigger denominator.

Misconception: Assumes a bigger denominator means a bigger fraction, without rewriting either fraction to check.

Next move: Have the student find a common denominator for both fractions before answering, every time, until the habit is automatic.

2. Simplify: $\frac{12}{18}$

Correct answer: $\frac{2}{3}$ (GCF = 6).

Likely wrong answer: $\frac{6}{9}$, from dividing by 2 and stopping.

Misconception: Divides by the first common factor found instead of the GCF, and doesn't check the result for a remaining common factor.

Next move: After simplifying, always check the result once more: does any number bigger than 1 still divide both parts?

Answer Key — Worksheets (Full Diagnostic)

Worksheet A: Guided Practice

1. In $\frac{4}{7}$, identify the numerator and the denominator.

1. Numerator: 4. Denominator: 7. Likely wrong answer: Numerator: 7, Denominator: 4 (swaps top and bottom). Misconception: The top and bottom positions of a fraction are reversed when naming them.

2. In $\frac{8}{13}$, identify the numerator and the denominator.

2. Numerator: 8. Denominator: 13. Likely wrong answer: Numerator: 13, Denominator: 8 (swaps top and bottom). Misconception: The top and bottom positions of a fraction are reversed when naming them.

3. $\frac{1}{6} = \frac{?}{24}$

3. $\frac{4}{24}$ (multiplied by 4). Likely wrong answer: $\frac{4}{28}$ (multiplies only the denominator). Misconception: Only one part of the fraction is scaled to reach the target denominator.

4. $\frac{5}{8} = \frac{?}{56}$

4. $\frac{35}{56}$ (multiplied by 7). Likely wrong answer: $\frac{5}{56}$ (multiplies only the denominator). Misconception: Only one part of the fraction is scaled to reach the target denominator.

5. Simplify: $\frac{6}{8}$

5. $\frac{3}{4}$. Likely wrong answer: $\frac{3}{8}$ (divides only the numerator). Misconception: Only one part of the fraction is divided when simplifying.

6. Simplify: $\frac{10}{15}$

6. $\frac{2}{3}$. Likely wrong answer: $\frac{5}{15}$ (divides only the numerator). Misconception: Only one part of the fraction is divided when simplifying.

7. Compare: $\frac{3}{7}$ and $\frac{4}{7}$

7. $\frac{3}{7} < \frac{4}{7}$. Likely wrong answer: $\frac{3}{7} > \frac{4}{7}$ (compares as if the larger numerator makes a smaller fraction). Misconception: The comparison direction is reversed even though the denominators already match.

8. Compare: $\frac{5}{9}$ and $\frac{5}{6}$

8. $5/9 < 5/6$. Likely wrong answer: $5/9 > 5/6$ (assumes the larger denominator gives the larger fraction). Misconception: A larger denominator is mistaken for a larger fraction instead of smaller pieces.

9. Order from least to greatest: $1/4$, $3/8$, $1/2$

9. $1/4$, $3/8$, $1/2$. Likely wrong answer: $1/2$, $3/8$, $1/4$ (orders by denominator size instead of value). Misconception: Fractions are ordered by looking at the denominators alone instead of comparing actual value.

10. Write $9/2$ as a mixed number.

10. $4 \frac{1}{2}$. Likely wrong answer: $9 \frac{1}{2}$ (uses the numerator as the whole number instead of dividing). Misconception: The numerator is used directly as the whole-number part instead of finding how many whole denominators fit into it.

11. Write $2 \frac{1}{5}$ as an improper fraction.

11. $11/5$. Likely wrong answer: $2/5 + 1$ written incorrectly as $3/5$ (adds instead of multiplying the whole number by the denominator). Misconception: The whole number is added to the denominator instead of multiplied by it before adding the numerator.

12. Is $3/7$ equal to, less than, or greater than 1?

12. Less than 1. Likely wrong answer: Greater than 1 (assumes any fraction with a numerator close to the denominator exceeds 1). Misconception: A fraction is judged against 1 by the size of its numerator alone rather than comparing it to the denominator.

13. Is $8/5$ equal to, less than, or greater than 1?

13. Greater than 1. Likely wrong answer: Less than 1 (misses that the numerator is larger than the denominator). Misconception: A fraction with numerator larger than denominator is not recognized as exceeding 1.

14. Order from least to greatest: $2/3$, $5/6$, $1/2$

14. $1/2$, $2/3$, $5/6$. Likely wrong answer: $5/6$, $2/3$, $1/2$ (orders by numerator size instead of value). Misconception: Fractions are ordered by looking at the numerators alone instead of comparing actual value.

15. Explain why $2/3 = 4/6$.

15. Multiplying both parts of $\frac{2}{3}$ by 2 gives $\frac{4}{6}$. Likely wrong answer: multiplies only one part of $\frac{2}{3}$ to check. Misconception: Only one part of the fraction is scaled when verifying equivalence, instead of both.

Worksheet B: Independent Practice

1. In $\frac{6}{13}$, identify the numerator and the denominator.

1. Numerator: 6. Denominator: 13. Likely wrong answer: Numerator: 13, Denominator: 6 (swaps top and bottom). Misconception: The top and bottom positions of a fraction are reversed when naming them.

2. $\frac{2}{3} = \frac{?}{15}$

2. $\frac{10}{15}$ (multiplied by 5). Likely wrong answer: $\frac{2}{15}$ (multiplies only the denominator). Misconception: Only one part of the fraction is scaled to reach the target denominator.

3. Simplify: $\frac{9}{12}$

3. $\frac{3}{4}$. Likely wrong answer: $\frac{9}{6}$ (divides only the denominator). Misconception: Only one part of the fraction is divided when simplifying.

4. Compare: $\frac{4}{5}$ and $\frac{7}{10}$

4. $\frac{4}{5} > \frac{7}{10}$. Likely wrong answer: $\frac{4}{5} < \frac{7}{10}$ (assumes the larger denominator gives the larger fraction). Misconception: A larger denominator is mistaken for a larger fraction instead of smaller pieces.

5. In $\frac{11}{15}$, identify the numerator and the denominator.

5. Numerator: 11. Denominator: 15. Likely wrong answer: Numerator: 15, Denominator: 11 (swaps top and bottom). Misconception: The top and bottom positions of a fraction are reversed when naming them.

6. Simplify: $\frac{14}{35}$

6. $\frac{2}{5}$. Likely wrong answer: $\frac{2}{7}$ (divides top and bottom by different factors). Misconception: Different numbers are used to divide the numerator and denominator instead of the same common factor.

7. $\frac{1}{4} = \frac{?}{36}$

7. $\frac{9}{36}$ (multiplied by 9). Likely wrong answer: $\frac{4}{36}$ (multiplies only the numerator). Misconception: Only one part of the fraction is scaled to reach the target denominator.

8. Compare: $\frac{5}{12}$ and $\frac{1}{3}$

8. $\frac{5}{12} > \frac{1}{3}$. Likely wrong answer: $\frac{5}{12} < \frac{1}{3}$ (assumes the larger denominator gives the larger fraction). Misconception: A larger denominator is mistaken for a larger fraction instead of smaller pieces.

9. Simplify: $\frac{21}{28}$

9. $\frac{3}{4}$. Likely wrong answer: $\frac{3}{7}$ (divides top and bottom by different factors). Misconception: Different numbers are used to divide the numerator and denominator instead of the same common factor.

10. Order from least to greatest: $\frac{3}{10}$, $\frac{2}{5}$, $\frac{1}{2}$

10. $\frac{3}{10}$, $\frac{2}{5}$, $\frac{1}{2}$. Likely wrong answer: $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{10}$ (orders by denominator size instead of value). Misconception: Fractions are ordered by looking at the denominators alone instead of comparing actual value.

11. Compare: $\frac{7}{8}$ and $\frac{11}{12}$

11. $\frac{7}{8} < \frac{11}{12}$. Likely wrong answer: $\frac{7}{8} > \frac{11}{12}$ (assumes the larger denominator gives the larger fraction). Misconception: A larger denominator is mistaken for a larger fraction instead of smaller pieces.

12. Simplify: $\frac{8}{20}$

12. $\frac{2}{5}$. Likely wrong answer: $\frac{4}{10}$ (divides top and bottom by different factors). Misconception: Different numbers are used to divide the numerator and denominator instead of the same common factor.

13. Write $\frac{23}{6}$ as a mixed number.

13. $3 \frac{5}{6}$. Likely wrong answer: $3 \frac{6}{6}$ left unsimplified, or $4 \frac{5}{6}$ miscounted. Misconception: The number of whole denominators is miscounted when converting from an improper fraction.

14. Compare: $\frac{2}{9}$ and $\frac{1}{4}$

14. $\frac{2}{9} < \frac{1}{4}$. Likely wrong answer: $\frac{2}{9} > \frac{1}{4}$ (assumes the smaller denominator gives the larger fraction without checking). Misconception: A comparison is made from denominator size alone without actually finding a common denominator.

15. Order from least to greatest: $\frac{5}{6}$, $\frac{7}{9}$, $\frac{2}{3}$

15. $\frac{2}{3}$, $\frac{7}{9}$, $\frac{5}{6}$. Likely wrong answer: $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{9}$ (orders by numerator size instead of value). Misconception: Fractions are ordered by looking at the numerators alone instead of comparing actual value.

16. Write $\frac{19}{4}$ as a mixed number.

16. $4\frac{3}{4}$. Likely wrong answer: $4\frac{4}{4}$ left unsimplified, or $4\frac{2}{4}$ miscounted. Misconception: The number of whole denominators is miscounted when converting from an improper fraction.

17. Write $4\frac{2}{9}$ as an improper fraction.

17. $\frac{38}{9}$. Likely wrong answer: $\frac{8}{9}$ (adds the whole number to the numerator instead of multiplying by the denominator first). Misconception: The whole number is added to the numerator directly instead of multiplied by the denominator before adding.

18. Write $3\frac{5}{8}$ as an improper fraction.

18. $\frac{29}{8}$. Likely wrong answer: $\frac{8}{8}$ (adds the whole number to the numerator instead of multiplying by the denominator first). Misconception: The whole number is added to the numerator directly instead of multiplied by the denominator before adding.

Worksheet C: Mixed Application

1. A student says $\frac{5}{9}$ is greater than $\frac{3}{4}$ because $5 > 3$. Explain the mistake.

1. $\frac{5}{9} < \frac{3}{4}$ because $\frac{20}{36} < \frac{27}{36}$. Likely wrong answer: $\frac{5}{9} > \frac{3}{4}$ because $5 > 3$. Misconception: Numerators are compared without accounting for different-sized parts. Next move: Convert both fractions to a common denominator.

2. Which recipe uses more milk: $\frac{3}{4}$ cup or $\frac{5}{8}$ cup? Show your method.

2. $\frac{3}{4}$ cup. Common denominator 8: $\frac{6}{8}$ vs $\frac{5}{8}$. Likely wrong answer: $\frac{5}{8}$ cup (assumes the larger denominator means more milk). Misconception: A larger denominator is mistaken for a larger amount instead of smaller pieces.

3. A student says $\frac{8}{12}$ is already simplified. Explain the mistake.

3. $\frac{8}{12}$ is not fully simplified; divide both terms by 4 to get $\frac{2}{3}$. Likely wrong answer: accepts $\frac{8}{12}$ as simplest form. Misconception: A common factor greater than 1 remains. Next move: Find the GCF before dividing.

4. Write three fractions equivalent to $\frac{2}{5}$.

4. Example: $\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$ (any multiples of both parts by the same number). Likely wrong answer: gives fractions that aren't actually equivalent (scales top and bottom by different amounts). Misconception: Different numbers are used to scale the numerator and denominator instead of the same common factor.

5. Write $\frac{17}{3}$ as a mixed number.

5. $5\frac{2}{3}$. Likely wrong answer: $3\frac{8}{3}$ left unsimplified, or the whole number miscounted. Misconception: The number of whole denominators is miscounted when converting from an improper fraction.

6. Write $5\frac{3}{4}$ as an improper fraction.

6. $\frac{23}{4}$. Likely wrong answer: $\frac{15}{4}$ (adds the whole number to the numerator instead of multiplying by the denominator first). Misconception: The whole number is added to the numerator directly instead of multiplied by the denominator before adding.

7. Which is closer to 1: $\frac{7}{8}$ or $\frac{5}{6}$? Explain.

7. $\frac{7}{8}$ is closer to 1. Likely wrong answer: $\frac{5}{6}$ is closer to 1 (compares numerators alone without finding actual distance). Misconception: Closeness to a benchmark is judged by the size of the numerator alone instead of by finding the actual gap.

8. Order from least to greatest: $\frac{3}{5}$, $\frac{7}{10}$, $\frac{1}{2}$, $\frac{4}{5}$

8. $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{4}{5}$. Likely wrong answer: orders by denominator size instead of finding a common denominator. Misconception: Fractions are ordered by looking at the denominators alone instead of comparing actual value.

9. Is $\frac{11}{8}$ greater than 1? Explain.

9. Yes. The numerator (11) is bigger than the denominator (8). Likely wrong answer: No (misses that the numerator exceeds the denominator). Misconception: A fraction with numerator larger than denominator is not recognized as exceeding 1.

10. Are $\frac{6}{9}$ and $\frac{2}{3}$ the same amount? Explain.

10. Yes. Both simplify to the same value. Likely wrong answer: says no without actually simplifying both. Misconception: Two fractions are judged unequal by appearance alone without simplifying to check.

11. Give a fraction between $\frac{3}{5}$ and 1.

11. Example: $\frac{4}{5}$ (any fraction strictly between $\frac{3}{5}$ and 1 works). Likely wrong answer: gives a fraction that is not actually between the two values. Misconception: A value is chosen without checking that it genuinely falls between the two given fractions.

12. Give an improper fraction equal to 4.

12. Example: $\frac{12}{3}$ (any fraction equivalent to 4 works). Likely wrong answer: gives a fraction where the numerator is not actually a multiple of the denominator by 4. Misconception: A fraction is proposed without checking it truly equals the required whole number.

13. Explain why $\frac{5}{6}$ and $\frac{15}{18}$ are equivalent.

13. Multiplying both parts of $\frac{5}{6}$ by 3 gives $\frac{15}{18}$. Likely wrong answer: multiplies only one part of $\frac{5}{6}$ to check. Misconception: Only one part of the fraction is scaled when verifying equivalence, instead of both.

14. Compare $\frac{5}{8}$ and $\frac{7}{12}$ using two different methods.

14. $\frac{5}{8}$ is greater. Common denominator 24: $\frac{15}{24}$ vs $\frac{14}{24}$. A benchmark check against one-half gives the same result. Likely wrong answer: uses two methods that don't actually agree, without noticing the mismatch. Misconception: Two different comparison methods are used without checking that they produce the same conclusion.

15. Explain why adding the same number to the numerator and denominator usually does not make an equivalent fraction.

15. Equivalent fractions require multiplying or dividing both terms by the same nonzero factor. Adding the same number usually changes the value; for example, $\frac{1}{2}$ becomes $\frac{2}{3}$ after adding 1. Likely wrong answer: claims equal additions preserve equivalence. Misconception: Additive change is substituted for multiplicative scaling. Next move: Compare cross-products before and after the change.

Answer Key — Exit Ticket

1. Numerator: 7. Denominator: 11.

2. $\frac{2}{3}$.

3. $\frac{3}{8} < \frac{2}{5}$.