

Chapter 2 — Factors, Multiples, Primes, and Divisibility

Teacher Chapter-at-a-Glance

Materials

A multiplication chart, printed or drawn on the board. No other materials required.

Talk Prompt

Ask the class whether 91 is prime before teaching the method. Most will say yes because it's odd. Let that guess stand for a moment before showing $91 = 7 \times 13$. The surprise is the lesson.

If Ready-to-Learn Reveals a Gap

If two or more Ready-to-Learn items reveal a gap, stop and rebuild multiplication and division fluency before moving into factors. Every skill in this chapter depends on being able to test division quickly.

Mastery Decision Guide

What a missed Tripwire item actually means, and what to do about it before moving on.

Missed Tripwire #1 → Assuming every odd number is prime (Mistake 2). *A prime classification for a number without showing any divisibility test beyond "it's odd" signals this error. Review before the next chapter.*

Missed Tripwire #2 → Picking GCF when the situation needs LCM, or the reverse (Mistake 5). *GCF used for a repeating-event problem, or LCM used for an equal-grouping problem, signals this error. Review before the next chapter.*

Ready-to-Learn Check

Quick check before starting. A wrong answer below points to a specific gap worth closing first, not just a miss to shrug off.

1. What is 7×6 ?

Correct answer: 42

Likely wrong answer: 36

Misconception: Confuses a nearby multiplication fact (6×6) with the one asked.

Next move: Have the student build the fact from a known anchor: $7 \times 5 = 35$, then add one more 7.

2. Does 3 go into 18 evenly? (Is 18 divisible by 3?)

Correct answer: Yes, $18 \div 3 = 6$.

Likely wrong answer: No, or an unchecked guess.

Misconception: Doesn't know how to test evenly dividing without long division, so guesses instead of checking.

Next move: Have the student actually divide $18 \div 3$ and confirm the remainder is 0.

3. Is 15 an even or odd number?

Correct answer: odd

Likely wrong answer: even

Misconception: Confuses odd and even, or uses a rule that doesn't apply, such as thinking numbers ending in 5 are even.

Next move: Have the student check whether the number splits evenly into two equal whole-number groups.

Common Mistakes

Students often run into the same kinds of trouble with this topic. Recognizing these patterns helps avoid them.

Comparing 0.86 and 0.809 by Place Value

	Ones		Tenths	Hundredths	Thousandths
0.86	0	.	8	6	0
0.809	0	.	8	0	9

Ones tie. Tenths tie. Hundredths decide it: $6 > 0$, so $0.86 > 0.809$.

The factors 18 and 24 share, with the greatest one circled as the GCF.

Mistake 1: Confusing factors and multiples

What students do: Listing multiples when factors were asked for, or the reverse.

Why this happens: Both words describe a multiplying relationship between two numbers, and the direction (into a number, or out from a number) is easy to lose track of.

What to do instead: Factors divide INTO a number and stop, a short list. Multiples come OUT from a number by counting up, an endless list.

Diagnostic trigger: A list that keeps growing past the number itself, offered as a list of factors, signals this exact error.

Mistake 2: Assuming every odd number is prime

What students do: Classifying a number as prime just because it is odd and doesn't obviously divide by 5.

Why this happens: Ruling out 2 (the only even prime) feels like it should be most of the work, but every other prime up to the square root still needs to be checked.

What to do instead: Test every prime up to the square root of the number: 2, 3, 5, 7, and so on. Stop only once none of them divide evenly.

Diagnostic trigger: A prime classification for a number without showing any divisibility test beyond "it's odd" signals this error.

Mistake 3: Calling 1 prime

What students do: Classifying 1 as a prime number.

Why this happens: 1 doesn't obviously fit "composite" either, so prime can feel like the only option left.

What to do instead: Prime means exactly two factors. 1 has exactly one factor, itself. That makes it neither prime nor composite, a category of its own.

Diagnostic trigger: 1 labeled as prime, rather than as neither, signals this error.

Mistake 4: Forgetting the matching factor in a pair

What students do: Listing a factor without its partner, so the factor list comes out incomplete.

Why this happens: It's easy to find one number that divides evenly and move on, without checking what it pairs with.

What to do instead: Every factor arrives in a pair. The moment a factor is found, divide the original number by it to get the matching partner, and record both.

Diagnostic trigger: A factor list missing a partner (for example, 4 is listed but $28 \div 4 = 7$ is not) signals this error.

Mistake 5: Picking GCF when the situation needs LCM, or the reverse

What students do: Choosing the wrong operation for a word problem: computing LCM for a splitting-into-groups situation, or GCF for a cycles-lining-up situation.

Why this happens: Both operations start from the same two numbers and the same factor relationship, so it's tempting to pick based on a memorized keyword instead of reasoning about the situation.

What to do instead: Ask what the problem is actually doing. Splitting something into the largest equal pieces or groups means GCF. Waiting for two repeating events to land on the same point again means LCM.

Diagnostic trigger: GCF used for a repeating-event problem, or LCM used for an equal-grouping problem, signals this error.

Practice

Teacher solutions follow this practice set.

1. List the factors of 45.
2. List the first 6 multiples of 7.
3. Is 37 prime, composite, or neither?
4. Is 51 prime, composite, or neither?
5. Is 2,508 divisible by 3?
6. Is 4,110 divisible by 5?
7. Find the GCF of 30 and 45.
8. Find the LCM of 4 and 10.
9. You have 32 pencils and 48 erasers. What is the greatest number of identical kits you can make using all of them?
10. Two joggers passing the start line every 6 minutes and 10 minutes. If they start together, how many minutes until they do it together again?

Practice Solutions

Use these concise solutions to check the method as well as the final answer.

1. List the factors of 45.

Solution: Factor pairs are 1×45 , 3×15 , and 5×9 , so the factors are 1, 3, 5, 9, 15, 45.

2. List the first 6 multiples of 7.

Solution: The first six positive multiples are 7, 14, 21, 28, 35, 42.

3. Is 37 prime, composite, or neither?

Solution: 37 is prime; its only positive factors are 1 and 37.

4. Is 51 prime, composite, or neither?

Solution: 51 is composite because $5 + 1 = 6$, which is divisible by 3; $51 = 3 \times 17$.

5. Is 2,508 divisible by 3?

Solution: Yes. The digit sum is $2 + 5 + 0 + 8 = 15$, and 15 is divisible by 3.

6. Is 4,110 divisible by 5?

Solution: Yes. A whole number ending in 0 or 5 is divisible by 5.

7. Find the GCF of 30 and 45.

Solution: The common factors include 1, 3, 5, and 15, so $\text{GCF}(30, 45) = 15$.

8. Find the LCM of 4 and 10.

Solution: Multiples of 4 are 4, 8, 12, 16, 20; multiples of 10 are 10, 20. Therefore $\text{LCM}(4, 10) = 20$.

9. You have 32 pencils and 48 erasers. What is the greatest number of identical kits you can make using all of them?

Solution: $\text{GCF}(32, 48) = 16$, so 16 identical kits can be made. Each kit has 2 pencils and 3 erasers.

10. Two joggers passing the start line every 6 minutes and 10 minutes. If they start together, how many minutes until they do it together again?

Solution: $\text{LCM}(6, 10) = 30$, so the joggers pass the start line together again after 30 minutes.

Tripwire Check

These questions test whether the thinking is solid — not just whether the procedure was memorized. Each wrong answer below points to a specific misconception, not just "try again."

1. Is 87 prime?

Correct answer: No. $87 = 3 \times 29$, so it's composite.

Likely wrong answer: Yes, because it's odd and doesn't end in 0, 2, 4, 5, 6, or 8.

Misconception: The odd-numbers-are-prime assumption, or stopping the divisibility test after only the most obvious rules (2, 5, 10) without testing 3.

Next move: Have the student test every prime up to the square root: 2, 3, 5, 7 for any number under 100, not just the "obvious" ones.

2. You need to cut 18 m and 24 m of ribbon into the longest equal pieces possible, with none left over. How long should each piece be?

Correct answer: 6 m, the GCF of 18 and 24.

Likely wrong answer: 72 m, the LCM of 18 and 24, from picking the wrong operation.

Misconception: Selects GCF or LCM by keyword pattern-matching instead of reasoning about whether the situation splits into equal pieces (GCF) or waits for two cycles to line up (LCM).

Next move: Ask what the problem is actually doing: splitting something into pieces, or waiting for two cycles to align. That determines GCF versus LCM.

Answer Key — Worksheets (Full Diagnostic)

Worksheet A: Guided Practice

1. List the factors of 24.

1. 1, 2, 3, 4, 6, 8, 12, 24. Likely wrong answer: 8, 16, 24, 48 (lists multiples instead of factors). Misconception: The direction of the multiplying relationship is reversed, giving numbers that count out from 24 instead of divide into it.

2. List the first 5 multiples of 4.

2. 4, 8, 12, 16, 20. Likely wrong answer: 1, 2, 3, 4 (lists factors instead of multiples). Misconception: The direction of the multiplying relationship is reversed, giving numbers that divide into 4 instead of counting out from it.

3. Is 23 prime, composite, or neither?

3. Prime. Likely wrong answer: Composite (assumes any odd number greater than 1 must be prime, or the reverse error of overtesting). Misconception: A primality decision is made without actually testing whether any number besides 1 and itself divides evenly.

4. Is 33 prime, composite, or neither?

4. Composite (3×11). Likely wrong answer: Prime (stops checking after ruling out 2 and 5). Misconception: The divisibility test stops too early, missing a factor like 3 that isn't immediately obvious.

5. Is 11 prime, composite, or neither?

5. Prime. Likely wrong answer: Composite (assumes every number greater than 10 must have more than two factors). Misconception: A number is assumed composite based on its size alone rather than by actually testing its factors.

6. Is 96 divisible by 2?

6. Yes. Likely wrong answer: No (misreads the divisibility rule for 2). Misconception: The last digit is checked against the wrong divisibility rule.

7. Is 96 divisible by 3?

7. Yes. Likely wrong answer: No (adds the digits but misapplies the rule for 3). Misconception: The digit sum is found but not correctly checked against the divisibility rule for 3.

8. Is 150 divisible by 5?

8. Yes. Likely wrong answer: No (misreads the divisibility rule for 5). Misconception: The last digit is checked against the wrong divisibility rule.

9. Find the GCF of 10 and 15.

9. 5. Likely wrong answer: 30 (finds the LCM instead of the GCF). Misconception: The two operations are confused, giving the smallest common multiple instead of the greatest common factor.

10. Find the GCF of 16 and 20.

10. 4. Likely wrong answer: 80 (finds the LCM instead of the GCF). Misconception: The two operations are confused, giving the smallest common multiple instead of the greatest common factor.

11. Find the LCM of 4 and 6.

11. 12. Likely wrong answer: 2 (finds the GCF instead of the LCM). Misconception: The two operations are confused, giving the greatest common factor instead of the smallest common multiple.

12. Find the LCM of 3 and 8.

12. 24. Likely wrong answer: 1 (finds the GCF instead of the LCM). Misconception: The two operations are confused, giving the greatest common factor instead of the smallest common multiple.

13. You have 20 markers and 30 stickers. What is the greatest number of identical bags you can make using all of them?

13. 10 bags. Likely wrong answer: 50 bags (adds the two quantities instead of finding their GCF). Misconception: An equal-grouping situation is solved by combining the numbers instead of finding their greatest common factor.

14. List the first 5 multiples of 5.

14. 5, 10, 15, 20, 25. Likely wrong answer: 1, 2, 3, 4, 5 (lists factors instead of multiples). Misconception: The direction of the multiplying relationship is reversed, giving numbers that divide into 5 instead of counting out from it.

15. List the factors of 50.

15. 1, 2, 5, 10, 25, 50. Likely wrong answer: 1, 2, 5, 10, 25, 50, 100 (includes a number that isn't actually a factor). Misconception: A factor list is built without checking that every listed number actually divides evenly.

Worksheet B: Independent Practice

1. List the factors of 54.

1. 1, 2, 3, 6, 9, 18, 27, 54. Likely wrong answer: misses a factor pair, e.g. omits 9 or 6. Misconception: A factor list is built by guessing rather than systematically checking factor pairs from 1 upward.

2. List the first 6 multiples of 9.

2. 9, 18, 27, 36, 45, 54. Likely wrong answer: 9, 18, 27, 36, 45 (stops one multiple short). Misconception: The count of multiples requested is miscounted, stopping before the full list is reached.

3. Is 29 prime, composite, or neither?

3. Prime. Likely wrong answer: Composite (assumes any two-digit odd number must be composite). Misconception: A primality decision is made without actually testing whether any number besides 1 and itself divides evenly.

4. Is 522 divisible by 3?

4. Yes. Likely wrong answer: No (adds the digits but misapplies the rule for 3). Misconception: The digit sum is found but not correctly checked against the divisibility rule for 3.

5. List the factors of 63.

5. 1, 3, 7, 9, 21, 63. Likely wrong answer: misses a factor pair, e.g. omits 7. Misconception: A factor list is built by guessing rather than systematically checking factor pairs from 1 upward.

6. Is 57 prime, composite, or neither?

6. Composite (3×19). Likely wrong answer: Prime (assumes any number not obviously even or ending in 5 must be prime). Misconception: A primality decision is made without actually testing whether any number besides 1 and itself divides evenly.

7. List the first 6 multiples of 13.

7. 13, 26, 39, 52, 65, 78. Likely wrong answer: 13, 26, 39, 52, 65 (stops one multiple short). Misconception: The count of multiples requested is miscounted, stopping before the full list is reached.

8. Is 936 divisible by 4?

8. Yes. Likely wrong answer: No (misapplies the divisibility rule for 4, checking only the last digit). Misconception: The rule for divisibility by 4 (checking the last two digits) is replaced with the rule for 2.

9. Is 61 prime, composite, or neither?

9. Prime. Likely wrong answer: Composite (assumes any number in this range must have more than two factors). Misconception: A number is assumed composite based on its size alone rather than by actually testing its factors.

10. Find the GCF of 27 and 36.

10. 9. Likely wrong answer: 108 (finds the LCM instead of the GCF). Misconception: The two operations are confused, giving the smallest common multiple instead of the greatest common factor.

11. Is 2340 divisible by 8?

11. No. Likely wrong answer: Yes (misapplies the divisibility rule for 8). Misconception: The rule for divisibility by 8 is confused with the simpler rule for 2 or 4.

12. Is 1 prime, composite, or neither?

12. Neither (1 has only one factor, itself). Likely wrong answer: Prime (assumes every number without an obvious factor must be prime, forgetting the special case).

Misconception: The number 1 is not recognized as the specific exception with only one factor.

13. Find the LCM of 5 and 8.

13. 40. Likely wrong answer: 13 (finds the GCF instead of the LCM). Misconception: The two operations are confused, giving the greatest common factor instead of the smallest common multiple.

14. Is 6021 divisible by 9?

14. Yes. Likely wrong answer: No (adds the digits but misapplies the rule for 9). Misconception: The digit sum is found but not correctly checked against the divisibility rule for 9.

15. Find the GCF of 40 and 60.

15. 20. Likely wrong answer: 240 (finds the LCM instead of the GCF). Misconception: The two operations are confused, giving the smallest common multiple instead of the greatest common factor.

16. Find the LCM of 9 and 12.

16. 36. Likely wrong answer: 3 (finds the GCF instead of the LCM). Misconception: The two operations are confused, giving the greatest common factor instead of the smallest common multiple.

17. Find the LCM of 6 and 14.

17. 42. Likely wrong answer: 2 (finds the GCF instead of the LCM). Misconception: The two operations are confused, giving the greatest common factor instead of the smallest common multiple.

18. Find the GCF and LCM of 14 and 21.

18. GCF: 7. LCM: 42. Likely wrong answer: GCF and LCM swapped, or only one value found. Misconception: The two related quantities are confused with each other, or only one is calculated when both are requested.

Worksheet C: Mixed Application

1. A student lists 8, 16, 24 as the factors of 8. Explain the mistake.

1. The factors of 8 are 1, 2, 4, and 8; 8, 16, and 24 are multiples. Likely wrong answer: accepts the list as factors. Misconception: Factors and multiples are reversed. Next move: Test whether each number divides 8 evenly.

2. A student says 39 is prime because it is odd. Explain the mistake.

2. $39 = 3 \times 13$, so it's composite. Likely wrong answer: agrees 39 is prime because it's odd. Misconception: A primality decision is made without actually testing whether any number besides 1 and itself divides evenly.

3. You have 36 rulers and 54 protractors. What is the greatest number of identical kits you can make using all of them?

3. 18 kits. Likely wrong answer: 90 kits (adds the two quantities instead of finding their GCF). Misconception: An equal-grouping situation is solved by combining the numbers instead of finding their greatest common factor.

4. Two lighthouses flashing every 12 minutes and 16 minutes. If they start together, how many minutes until they do it together again?

4. 48 minutes. Likely wrong answer: 28 minutes (adds the two intervals instead of finding their LCM). Misconception: A repeating-event situation is solved by combining the numbers instead of finding their least common multiple.

5. Pack 60 muffins and 75 cookies into identical trays with nothing left over. What is the greatest number of trays you can make?

5. 15 trays. Likely wrong answer: 135 trays (adds the two quantities instead of finding their GCF). Misconception: An equal-grouping situation is solved by combining the numbers instead of finding their greatest common factor.

6. Is 8232 divisible by 6? Explain using divisibility rules.

6. Yes. 8,232 is even and its digit sum is 15, so it is divisible by both 2 and 3 and therefore by 6. Likely wrong answer: tests only divisibility by 2. Misconception: Both conditions for divisibility by 6 are not checked. Next move: Record the 2-test and 3-test separately before concluding.

7. Is 5481 divisible by 9? Explain using divisibility rules.

7. Yes. Digit sum = 18, which is divisible by 9, so 5481 is too. Likely wrong answer: No (finds the digit sum but misapplies the rule for 9). Misconception: The digit sum is found but not correctly checked against the divisibility rule for 9.

8. Give an example of an odd composite number and explain why it is composite.

8. Answers will vary. Example: $15 = 3 \times 5$, so it has factors besides 1 and itself. Likely wrong answer: gives a prime number as the example. Misconception: An odd number is assumed to be composite without actually checking for a factor besides 1 and itself.

9. Give an example of a two-digit prime number and explain why it is prime.

9. Answers will vary. Example: 41. No number from 2 to 6 (up to $\sqrt{41}$) divides evenly. Likely wrong answer: gives a composite number as the example. Misconception: A two-digit number is assumed to be prime without actually testing it against every prime up to its square root.

10. Find the GCF and LCM of 16 and 24.

10. GCF: 8. LCM: 48. Likely wrong answer: GCF and LCM swapped, or only one value found. Misconception: The two related quantities are confused with each other, or only one is calculated when both are requested.

11. Is the GCF always smaller than both numbers? Explain using 21 and 28 as an example.

11. Yes here: $\text{GCF}(21, 28) = 7$, which is smaller than both. This is generally true unless one number divides the other evenly. Likely wrong answer: says no without checking an actual example. Misconception: A general claim about GCF is answered without testing it against the specific numbers given.

12. Is the LCM always the product of the two numbers? Explain using 8 and 12 as an example.

12. No. $\text{LCM}(8, 12) = 24$, not 96 (the product). They only match when the GCF is 1. Likely wrong answer: says yes without checking an actual example. Misconception: A general claim about LCM is answered without testing it against the specific numbers given.

13. Pack 84 granola bars and 108 juice boxes into identical snack packs with nothing left over. Greatest number of snack packs?

13. 12 snack packs. Likely wrong answer: 192 snack packs (adds the two quantities instead of finding their GCF). Misconception: An equal-grouping situation is solved by combining the numbers instead of finding their greatest common factor.

14. Two buses leaving the depot every 15 and 20 minutes. When will they next happen at the same time?

14. After 60 minutes. Likely wrong answer: 35 minutes (adds the two intervals instead of finding their LCM). Misconception: A repeating-event situation is solved by combining the numbers instead of finding their least common multiple.

15. Explain why 0 is a multiple of 11 but not a factor of 11.

15. Zero is a multiple of 11 because $11 \times 0 = 0$. Zero is not a factor of 11 because $11 \div 0$ is undefined. Likely wrong answer: says zero is both. Misconception: The definitions of factor and multiple are being treated as interchangeable. Next move: Write one multiplication statement and one division statement.

Answer Key — Exit Ticket

1. 1, 2, 4, 5, 10, 20.

2. Composite ($3 \times 3 \times 7$).

3. 6.